MODULE – 4 : Electrical Properties of Materials

Syllabus:

Conductors: Quantum Free Electron Theory of Metals, Density of States, Fermi-energy, Fermi factor, Variation of Fermi factor with temperature, Electron concentration (qualitative discussion), Expression for electrical conductivity.

Semiconductors: Expression for the electron and hole concentrations, Expression for electrical conductivity in semiconductor.

Hall effect, expression for Hall coefficient.

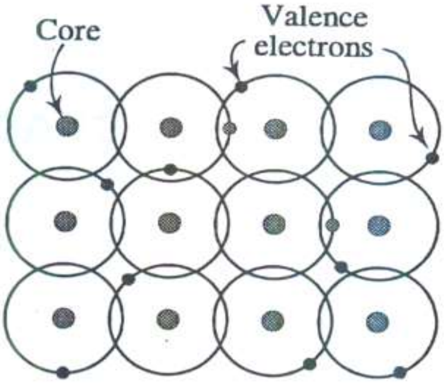
Dielectrics: Polarization and its types in dielectric materials, Internal field, Clausius – Mossotti equation, Numeral Problems.

CONDUCTORS

Free–electron concept

Metal is made up of atoms and these atoms consist of valence electrons, which are responsible for electrical conduction in the bulk state of the metal. **For e.g.:** A copper atom consists of 29 electrons out of which 28 electrons are bound in the 3 shells forms the core and one electron remaining in the fourth shell is the free valence electron. As a whole an atom is neutral.

When a large number of copper atoms join to form a metal the boundaries of the neighbouring atoms slightly overlap on each other. Due to this, the valence electrons find continuity from atom to atom and can move easily throughout the body of the metal, but are restricted within the boundaries of the solid.



Since each atom contributes equal number of electrons, there will be a very large number of electrons which are free in a metal. Such electrons are called as **free electrons** and they account for the bulk properties of the metal such as electrical conductivity, thermal conductivity etc. They are also called conduction electrons.

The disconnection of the valence electrons results in a virtual loss of a negative charge for that atom so that it becomes a positive ion. The array of such ions forms a three-dimensional structure called a **lattice.**

Under thermal equilibrium conditions, the free electrons in a solid are in a state of random motion. At temperature T, they possess an average kinetic energy given by $\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$

Where v_{th} is the mean **thermal velocity**.

Quantum Free Electron Theory of Metals

In 1928, Arnold Sommerfeld succeeded in overcoming many of the drawbacks of the classical free electron theory by using Pauli's exclusion principle and Fermi-Dirac statistics is known as quantum free electron theory. The following are the assumptions of quantum free electron theory.

The main assumptions of quantum free electron theory are:

1. The energy values of the conduction electrons are quantized. They can have only discrete energy values.
2. The distribution of electrons in the various allowed levels occur as per Pauli's exclusion principle which states that no two electrons can have same set of quantum numbers.
3. The distribution of energy among the free electrons is according to Fermi-Dirac statistics.
4. The electrons travel in a constant potential inside the metal but stay confined within its boundaries.
5. The attraction between the electrons and the lattice ions and the repulsion between the electrons are ignored.

Energy bands in solids

- The discrete energy level of an atom becomes bands during the formation of solid due to mutual influence of constituent atoms
- Each band consists of a large number of energy levels which correspond to a range of energy values.
- The bands are separated by certain gaps called **forbidden bands** or **energy band gaps**.
- The electron can occupy only those energy levels which lie within an energy band.
- Because of quantization rules there is a limit on the number of electrons which can fill an energy band.
- If an energy band contains the maximum quota of electrons, it is said to be filled band
- An empty band or partially filled band provides energy levels to which electrons can be energized from lower levels or bands.

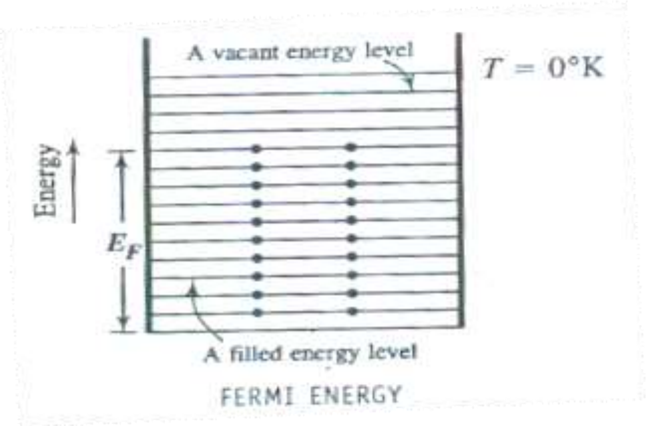
- In every energy band diagram, there will be a band called valence band below which all the bands are occupied and above which all the bands are empty.
- The empty band which is immediately above the valence is called conduction band.
- The gap between the valence band the conduction band is called the **forbidden band or energy gap**.

Fermi Energy:

There are ‘N’ allowed energy levels for free electrons in a metal which are separated by energy differences that are the characteristics of the material and are quantized. As per Pauli’s exclusion principle, each allowed energy level can accommodate a maximum of two electrons with opposite spin. The filling up of the electrons in the energy levels should be undertaken from the lowest energy level. So, in each energy level there are two electrons one with spin up and other with spin down. However there are still higher energy levels left vacant.

“The energy of the highest occupied energy level at zero degree absolute is called **Fermi energy (E_F)** and the energy level is referred as **Fermi level”**

At absolute zero temperature i.e. when the metal is not under the influence of any electrical field or thermal energy, no electrons will be present above the Fermi level. They are completely empty and those levels below Fermi level are completely filled.



Density of States

In a solid material, the permitted energy levels are in terms of bands. Each band is spread over an energy range of few eVs and the number of energy levels in each band is extremely large. Because of such distribution, the energy values appear to be virtually continuous over the band. A closer look reveals that the energy levels are not evenly distributed in the band. i.e. the density of energy levels in the band varies with the energy. This is realized through a function known as density of states function denoted as g (E). “It is the number of allowed energy levels per unit energy interval in the band associated with the material per unit volume".

The number of states in an energy range E and E+dE for the electrons in a 3- dimensional solid of unit volume or Density of states in the energy interval dE at 'E' is given by

$$g(E)dE = \left(\frac{8\sqrt{2}\pi m^{3/2}}{h^3}\right)E^{-1/2}dE$$

Fermi – Dirac statistics:

In a metal, there is large number of free electrons and each of them possesses an energy corresponding to the energy state of the valence band. Under thermal equilibrium, the free electrons acquire energy obeying the statistical rule known as Fermi – Dirac statistics. The electrons are identical particles and indistinguishable particles. Fermi – Dirac statistics permit the evaluation of the probability of finding electrons occupying energy levels in a certain energy range. This evaluation is done through a function called the Fermi factor.

Fermi Factor:

At temperatures above absolute zero, the material will be receiving thermal energy from the surroundings. So, the electrons just below the Fermi level absorb the thermal energy and will occupy the higher empty levels. The electrons occupying energy levels far below the Fermi level cannot absorb this energy because there are no unoccupied higher energy levels into which the electrons can come into, when their energies increase by small amount.

Though such excitations seem to be random the resulting distribution of electrons in various energy levels after excitation will be systematic. The distribution is governed by a statistical function when the system is in thermal equilibrium (steady state).

The probability 'f(E)' that a given energy state with energy 'E'is occupied at a steady temperature is given by

$$\boldsymbol{f(E) = \frac{1}{e^{\frac{E-E_F}{kT}}+1}}$$

Here f(E) is called Fermi factor.

"**Fermi factor** is defined as the probability of occupation of a given energy state for a material in thermal equilibrium".

Dependence of Fermi factor on temperature and effect on the occupancy of energy levels:

The probability ‘f(E)’ that a given energy state with energy ‘E’ is occupied at a steady temperature is given by

$$\boldsymbol{f(E) = \frac{1}{e^{\frac{E-E_F}{kT}}+1}}$$

The different cases of distribution are as follows;

CASE1: Probability of occupation for $E < E_F$ at $T = 0K$

When T = 0K and $E < E_F$, we have

$$f(E\ ) = \frac{1}{e^{-\infty}+1} = \frac{1}{0+1} = 1$$

∴ f(E) = 1, for $E < E_F$

It means that the energy level is certainly occupied and $E < E_F$ applies to all the energy levels below ‘E_F’.

CASE2: Probability of occupation for $E > E_F$ at $T = 0K$

When T = 0K and $E < E_F$, we have

$$f(E) = \frac{1}{e^{+\infty}+1} = \frac{1}{\infty+1} = 0$$

∴ f(E) = 0, for $E > E_F$

∴ At T = 0K, all the energy levels above the Fermi level are unoccupied.

Thus, at T = 0K, the variation of f(E) for different energy values, becomes a step function as shown in the figure below

CASE 3: Probability of occupation at ordinary temperature:

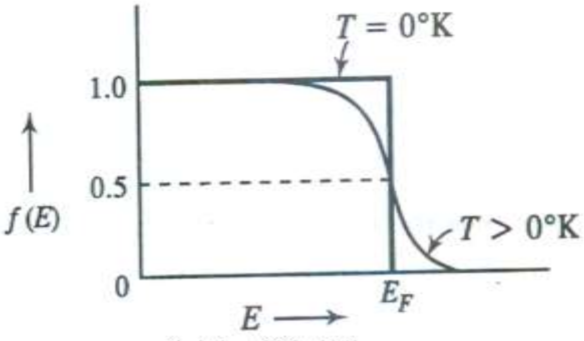
At ordinary temperatures f(E) remains 1 for $E << E_F$, and starts decreasing from 1 as ‘E’ becomes closer to E_F as sown in fig.

At $E = E_F$ we have,

$$f(E) = \frac{1}{e^0+1} = \frac{1}{1+1} = \frac{1}{2}$$

∴The value of f(E) becomes ½ at $E = E_F$

Thus, Fermi energy is the most probable energy or the average energy of the electrons which undergo transitions across the Fermi level at temperature above zero degree absolute.



Variation of f(E) with E

Electron concentration:

Metals, such as copper and aluminium, are held together by bonds that are very different from those of molecules. Rather than sharing and exchanging electrons, a metal is essentially held together by a system of free electrons that wander throughout the solid. The simplest model of a metal is the **free electron model.** This model views electrons as a gas. In a metal, due to the randomness in the direction of motion of the conduction electrons, the probability of finding an electron moving in any given direction is equal to finding some other electron moving in exactly the opposite direction in the absence of an electric field.

In the absence of external electric field, the motion of electrons in a metal will be random, because of this there is no drift but when external electric field is applied the electrons slowly drift in a direction opposite to the electric field.

$$n_e = \frac{m\sigma}{e^2\tau}$$

Where : n_e is the electron concentration
e is the electron charge
τ is the mean collision time
m is the mass of the electron
σ is the electric conductivity

Expression for electrical conductivity and resistivity based on quantum free electron theory

The momentum of the free electrons in the metal is given by

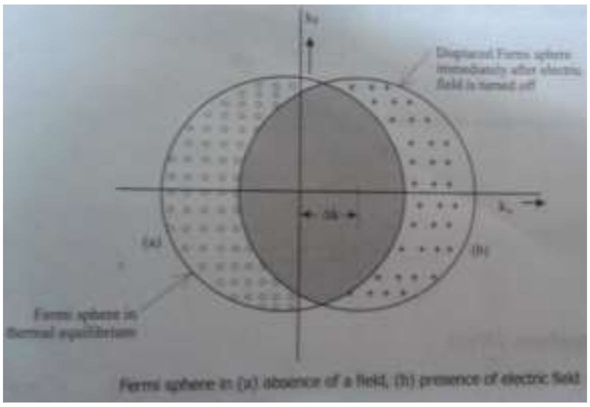
$$p = \frac{h}{\lambda} \quad ; \qquad \text{but} \qquad k = \frac{2\pi}{\lambda}$$

Hence, $$p = \frac{hk}{2\pi}$$

$$mv = \frac{hk}{2\pi}$$

$$v = \frac{hk}{2\pi m} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In thermal equilibrium, the free electrons are moving in random fashion. The velocities of these free electrons can be plotted in velocity space or k-space by assuming v_F (Fermi velocity) as maximum velocity. The resulting sphere having v_F as the radius is known as Fermi sphere/ Fermi surface. Only those electrons present near the Fermi surface can participate in electrical conduction. When no electric is applied, centre of the sphere coincides with the origin of the k-space so that the net momentum is zero.



Fermi sphere in (a) absence of a field, (b) presence of electric field

When an electric filed is applied, the electrons are accelerated in the direction of theforce. The acceleration of the electrons is given by

Differentiating (1) w.r.t. 't' we get,

$$\frac{dv}{dt} = \frac{h}{2\pi m}\frac{dk}{dt} \Rightarrow \quad m\frac{dv}{dt} = \frac{h}{2\pi}\frac{dk}{dt} \ldots\ldots\ldots..(2)$$

we know that Force applied on the electron in the applied field is

$$F = eE$$

$$ma = eE$$

$$m\frac{dv}{dt} = eE$$

Therefore,

$$\frac{h}{2\pi}\frac{dk}{dt} = eE \Rightarrow dK = \frac{2\pi eEdt}{h} \ldots\ldots\ldots\ldots (3)$$

If the field is applied for time period 't', then integrating the above equation from 0→t, and applying the limits we get

$$k(t) - k(0) = \frac{2\pi eEt}{h}$$

$$\Delta k = \frac{2\pi eEt}{h} \ldots\ldots\ldots\ldots..(4)$$

Thus, if the field is applied at time t=0 to filled Fermi sphere centred at the origin of the k-space, then in characteristic time , $\tau = \tau_F = \frac{\lambda_F}{v_F}$, the sphere might have moved to a new centre at

$$\Delta k = \frac{2\pi eE}{h}\frac{\lambda_F}{v_F} \ldots\ldots\ldots\ldots..(5)$$

The steady state current density is given by

$$J = ne\Delta v$$

$$\Delta v = \frac{h\Delta k}{2\pi m}$$

$$J = \left[\frac{neh\Delta k}{2\pi m^*}\right] \ldots\ldots\ldots\ldots\ldots..(6)$$

Here m* is the effective mass of the electrons. The electrical conductivity (σ) is related to current density by

$$J = \sigma E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

From equation (6) and (7)

$$\sigma = \frac{neh\Delta k}{2\pi m^* E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

Substituting from equation (5) in equation (8) we get

$$\sigma = \frac{neh}{2\pi m^* E}\left(\frac{2\pi eE}{h}\frac{\lambda_F}{v_F}\right)$$

$$\sigma = \frac{ne^2}{m^*}\left(\frac{\lambda_F}{v_F}\right)$$

Where m* is the effective mass of electrons.

Also, the expression for resistivity is given by $\rho = \frac{m^* v_F}{ne^2 \lambda}$

SEMICONDUCTORS

Conductivity of solids can be explained on the basis of band theory. The important energy bands in solids are

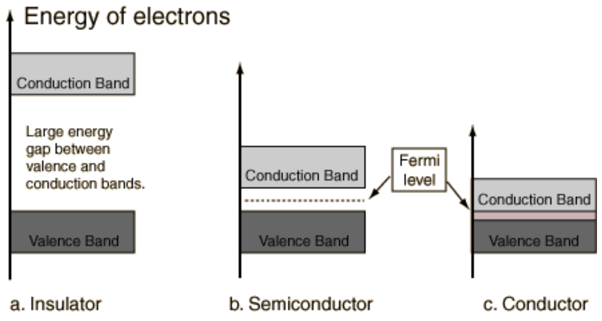
1. **Valence Band:** The electron in the outer most orbit of an atom is known as valence electron. The range of energies possessed by valence electrons is known asvalence band. The valence band may be completely or partially filled.

2. **Conduction Band:** The range of energies possessed by conduction electrons isknown as conduction band. In some metals, the valence electron may be loosely packed and may get detached to become free electron. These free electrons are responsible for conduction of current in a conductor and are hence called conduction electrons.

Significance of Band Gap *Eg*

The band gap energy Eg is minimum amount of energy required for breaking a covalent bond and to excite an electron from valence band to conduction band. The energy required to break a covalent bond in Ge is 0.72eV and for Si it is 1.12eV at 300K.

Forbidden energy band/gap: The separation between bottom of conduction band and top of valence band in the energy level diagram of solids is known as forbiddenenergy band. There are no electrons in the forbidden energy gap.



Classification of semiconductors:

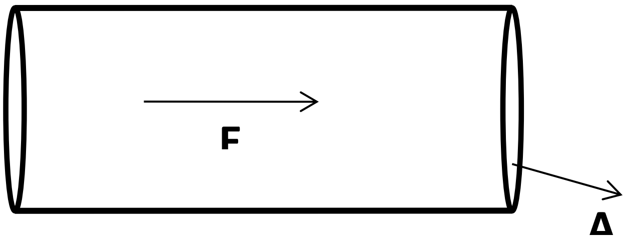
The materials having moderate electrical conductivity are called as semiconductors. These materials have a completely filled VB and completely filled CBat 0K. Semiconductors are classified into two types on the basis of concentrations of electrons and holes in the material.

1. Intrinsic or pure semiconductor
2. Extrinsic or doped semiconductor

Fermi Level in semiconductors:

In semiconductors, at Temperature, T = OK the conduction band (CB) is completely empty and the valence band (VB) is completely filled. But, at ordinary temperatures such as room temperatures, electrons get excited from the top of the valence band to conduction band and occupy energy levels at the bottom of the conduction band. As CB is a higher energy band, the electrons undergo de-excitation to VB. This process of excitation and de-excitation continues and the electron involved becomes conduction electrons. This means that the conduction electrons are distributed between the energy levels in the bottom of the CB and top of the VB. The average energy of all such energy levels is called Fermi level of the semiconductor. In the case of pure or intrinsic semiconductors, the Fermi level corresponds to level in the forbidden gap exactly in between CB and VB. Fermi level also signifies the average energy of the conduction electrons.

Expression for electrical conductivity of a semiconductor:



Consider a semiconductor of area cross section ‘A’. Let an electric field of ‘E’ is applied to the conductor. Let v_e and v_h be the drift velocities of electrons and holes respectively due to applied field E.

The current due to electron is

$$I_e = n_e e A v_e \rightarrow (1)$$

The current due to holes is

$$I_h = n_h e A v_h \rightarrow (2)$$

Here and n_e & n_h are the number of electrons and holes per unit volume.

The total current is given by

$$I = I_e + I_h = n_e e A v_e + n_h e A v_h$$

The total current is the sum of the current due electrons and holes. The total current density is given by

$$J = n_e e v_e + n_h e v_h \rightarrow (3)$$

By definition, mobility of charge carriers is the drift velocity per unit electric filed.

Therefore the drift velocity of electrons is

$$v_e = \mu_e E \qquad \mu_h = \frac{v_h}{E}$$

and the drift velocity of holes is

$$v_h = \mu_h E$$

where μ_e & μ_h are the motilities of electrons and holes respectively.

Substituting for v_e & v_h in equation (3) we get

$$J = n_e e \mu_e E + n_h e \mu_h E \rightarrow (4)$$

We have Ohm’s law $J = \sigma E \rightarrow (5)$

Where σ is conductivity, which is defined as the current density per unit applied electric field.

Comparing equation (4) and (5), we get

$$\sigma = n_e e \mu_e + n_h e \mu_h \rightarrow (6)$$

This equation gives total conductivity of a semiconductor.

For intrinsic semiconductors, $n_e = n_h = n_i$; n_i is the intrinsic carrier concentration. Hence

$$\sigma_i = n_i e(\mu_e + \mu_h)$$

Electron and hole concentration:

The number of electrons per unit volume in the conduction band of a semiconductor is known as **electron concentration**. Similarly, the number of holes per unit volume in the valence band of a semiconductor is known as **hole concentration**.

The electron concentration n_e is given by

$$\boldsymbol{n_e = \frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{3/2} e^{\left(\frac{E_F - E_g}{kT}\right)}}$$

The hole concentration n_h is given by

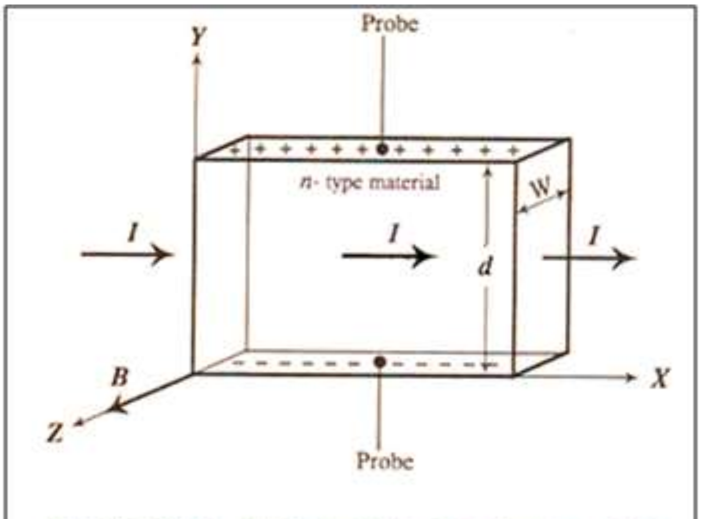
$$\boldsymbol{n_h = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}}$$

In the above equation, E_F is the Fermi energy of the semiconductor, E_g is the energy gap of the semiconductor, m_e^* and m_h^* are the effective masses of electrons and holes, T is the absolute temperature.

Hall Effect

If a material carrying current is placed in a transverse magnetic field, an electric field is produced in the material in a direction perpendicular to both the current and magnetic field. This phenomenon is called **Hall Effect**. The electric field generated is called **Hall field** and corresponding voltage is **Hall voltage**.

Expression for Hall coefficient :



GENERATION OF ELECTRIC FIELD DUE TO HALL EFFECT

Consider a rectangle slab of a semiconductor material in which a current ‘I’ is flowing in positive x-direction. Let the semiconductor material be of n type, which means that the charge carriers are electrons.

Let a magnetic field, **B** is applied along Z-direction as shown in the figure.
Under the influence of a magnetic field, the electrons experience the Lorentz-force, F_L given by
$F_L = -Bev \rightarrow (1)$; -ve is due to electron charge
Where ‘e ‘is the magnitude of charge on the electron and ‘v’ is drift velocity of the electrons.
Applying the Flemings left -hand rule, the force $\mathbf{F_L}$ is acting on the electron along the negative y-direction. The electrons are therefore deflected downwards resulting in decrease and increase in electron density at the upper and lower surfaces respectively. Hence a potential difference $\mathbf{V_H}$ called **Hall voltage** appears between the upper and lower surface of the semiconductor material. This voltage establishes an electric field E_H, called **Hall field** across the conductor in the negative y- direction.

The field $\mathbf{E_H}$, exerts an upward force $\mathbf{F_H}$ on the electrons given by,

$$F_H = -eE_H \rightarrow (2)$$

Now as the deflection of electrons continues in the downward direction due to Lorentz force $\mathbf{F_L}$, the Hall field increases. As a result, the force $\mathbf{F_H}$ which acts on the electron in upward direction also increases till it becomes equal to $\mathbf{F_L}$. Thus, at **equilibrium**,

$$F_L = F_H$$

$$\text{i.e.} -Bev = -eE_H \quad \text{since} \quad F_{\mathbf{L}} = evB$$

$$\Rightarrow E_H = Bv \rightarrow (3)$$

Using the relation, $I = nAev,$

The current through the semiconductor $I = nAev$

$$v = \frac{I}{Ane}$$

Substituting for v, in equation (3) we get

$$E_H = B\frac{I}{wdne} \text{------ (4)} \quad \text{since } \mathbf{A = wd}$$

The Hall voltage, $V_H = E_H d = \frac{BId}{wdne} = \frac{BI}{wne}$ -------- (5)

where **n** is charge carrier concentration.

$$V_H = \frac{BI}{\rho_{ch} w}$$

Where $\boldsymbol{\rho_{ch}} = \mathbf{ne}$, is the charge density which is a constant for a given material.

Hence $V_H = \frac{R_H B I}{w}$

where R_H is a constant called Hall coefficient, $R_H = \frac{1}{\rho_{ch}} = \frac{1}{ne}$

Thus by measuring V_H, I & w and by knowing B, the charge density ρ_{ch} can be determined using which Hall coefficient can be evaluated.

Importance of Hall Effect:

The importance of Hall Effect in the field of semiconductor is that it helps to determine

1) The type of semiconductor.
2) The sign of majority charge carriers.
3) The majority charge carrier concentration.
4) The mobility of majority charge carriers.
5) The mean drift velocity of majority charge carriers.

Dielectric Properties

Dielectrics

A dielectric is an electrically non-conducing material which provides electrical insulation between two media (conductors) which are at different potentials, and also serves as an electrical charge storage aid under certain circumstances.

E.g.: glass, porcelain, wood, rubber or waxed paper.

Electric dipole:

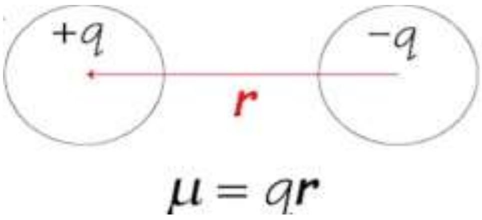
Electric dipole is a pair of equal and opposite charges, +q and −q separated by a very small distance

Examples of electric dipole: - Dipoles are common in nature. Molecules like H_2O, HCl, CH_3COOH are electric dipoles and have permanent dipole moments.

Dipole moment:

A dipole moment is the product of the magnitude of the charge and the distance between the centres of the positive and negative charges in a system. It is denoted by

$$\mu = qr$$



Units: C-m or Debye,

Polar and non-polar dielectric materials

Polar dielectric	**non-polar dielectric**
In polar material, the effective centers of the negative and positive charges in the molecules do not coincide with each other even in the absence of any external field	In non-polar materials, the effective center of the negative charge distribution coincides with the effective center of the positive charges
Eg : H_2O, HCl, CO	**Eg :** O_2, H_2, N_2

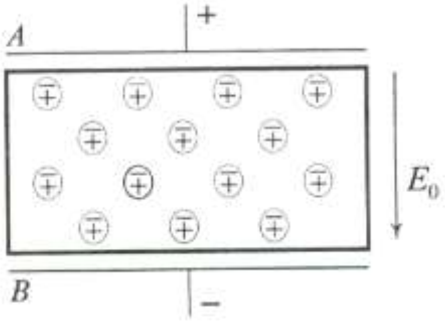
Polarization in dielectrics

The Process of formation of dipoles or alignment of already existing dipoles on the application of an electric field on the dielectric material is called polarization.

(i) Polarization in the case of non-polar dielectric:

Consider a rectangular slab of a non-polar dielectric material. Let the slab be placed between two electrodes A & B, between which a uniform electric field E is set up.

Under the influence of the electric field, the positive and negative charges of the various molecules in it experience pulling forces in opposite directions. As a result, the effective centers of positive and negative charges get separated till the restoring forces balance the forces due to the applied field. Due to the separation between the two charge centers, each molecule develops a dipole moment in the direction of the electric field. Such dipoles are produced throughout the solid.



POLARIZATION

Now, inside the body of the dielectric slab, though the polarization charges appear in every molecule, opposite charges in the neighboring molecules neutralize each other. This kind of cancellation takes place throughout the body of the material since there are equal amount of negative and positive charges. However, the charges which are at the end surfaces of the slab do not find the opposite kind of charges for cancellation as is evident from the figure. Thus the effect of the applied field is to cause the appearance of net opposing charges at the end faces of the slab.

(ii) Case of polar dielectric:

In the case of a polar dielectric material, there are permanent dipoles oriented randomly. Under the influence of the applied electric field, the molecular dipoles experience a torque which tends to align their dipole moments along the direction of the field. This tendency is opposed by the thermal agitation inside the material. Between these two opposing events, some alignment is achieved under sufficiently strong fields. Once the alignment is established, the surface charges appear at the ends of the slab in a way similar to the case of non-polar dielectric materials.

Types of polarization processes

There are three different mechanisms through which electrical polarization can occur in dielectric materials when they are subjected to an external electric field. Accordingly there are three main types of polarizations.

They are,

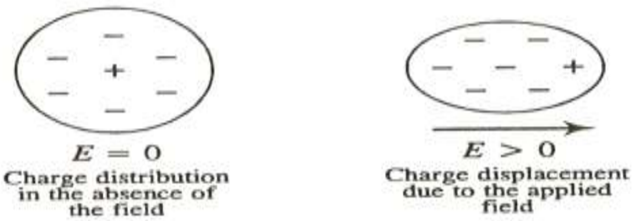
1) Electronic polarization,

2) Ionic polarization

3) Orientational polarization

1. Electronic polarization:

The electronic polarization occurs due to displacement of the positive and negative charges in a dielectric material owing to the application of an external electric field. The separation created between the charges leads to development of a dipole moment. This process occurs throughout the material. Thus the material as a whole will be polarized.



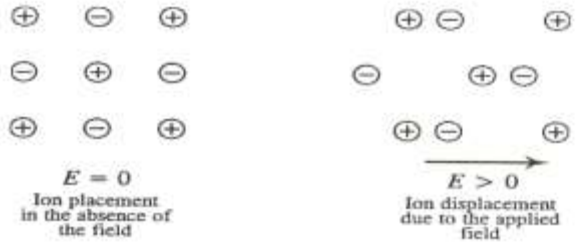
ELECTRONIC POLARIZATION

The electronic polarizability, α_e for a rare gas atom is given by,

$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$; where N is the number of atoms/unit volume.

2. Ionic Polarization:

Ionic polarization occurs only in those dielectric materials which possess ionic bonds such as in NaCl. When ionic solids are subjected to an external electric field, the adjacent ions of opposite sign undergo displacement. The displacement causes an increase or decrease in the distance of separation between the atoms depending upon the location of the ion pair in the lattice.

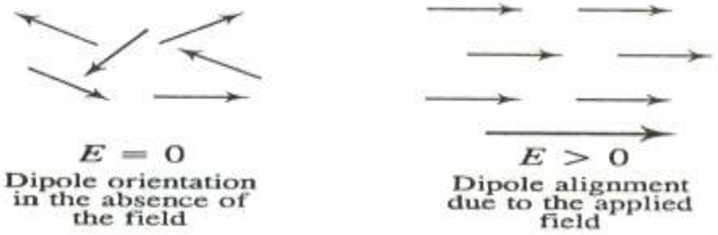


IONIC POLARIZATION

This results in a net dipole moment due to the shifting of electron clouds relative to the nuclei.

3. Orientational Polarization:

Orientational polarization occurs in those dielectric materials which possess molecules with permanent dipole moment (i.e., in polar dielectrics). The orientation of these molecules will be random normally due to thermal agitation. Because of randomness in orientation, the material has net zero dipole moment. But under the influence of an applied electric field, each of the dipoles undergo rotation so as to reorient along the direction of the field because of which, the overall cancellation of dipole moments due to randomness does not hold good any more (Fig. 7). Thus the material itself develops electrical polarization.



ORIENTATIONAL POLARIZATION

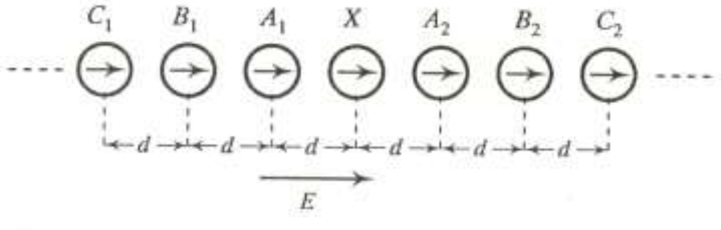
The orientational polarization is strongly temperature dependent and decreases with increase of temperature.

Internal field:

What is internal field? Write an expression for the internal field in case of solid and liquid dielectrics?

"**The internal field, or the local field**, is the electric field that acts at the site of any given atom of a solid or a liquid dielectric subjected to an external electric field. It is defined as **"the resultant of the applied field and the field due to all the surrounding dipoles**".

Consider a dielectric material either solid or liquid kept in an external uniform electric field of strength **E**. In the material let us consider an array of equidistant atomic dipoles arranged parallel to the direction of the field as shown in fig.



LINEAR ARRAY OF ATOMS IN AN ELECTRIC FIELD

Let the interatomic distance be 'd' and dipole moment of each of atomic dipole be μ. Then the internal field at the site of an atom is given by

$$E_i = E + \frac{1.2\mu}{\pi\varepsilon_0 d^3}$$

If α_e is the electronic polarizability of the atoms, we can write,

$$\mu = \alpha_e E_i$$

Hence, $E_i = E + \frac{1.2\alpha_e E_i}{\pi\varepsilon_0 d^3}$

Or

$$E_i = \frac{E}{1 - \frac{1.2\alpha_e}{\pi\varepsilon_0 d^3}}$$

In 3-dimensional cases, the general equation for internal field is expressed as,

$$E_i = E + \frac{\gamma P}{\varepsilon_0}$$

where, P is the dipole moment/unit volume for the material, and γ is the proportionality constant called internal field constant.

In the 3-dimensional case, if it is cubic lattice, then it can be shown that γ = 1/3 in which event, the internal field is named Lorentz field given by,

$$E_{Lorentz} = E + \frac{P}{3\varepsilon_0}$$

Above equation is known as Lorentz relation.

Clausius – Mossotti relation

Consider an elemental solid dielectric material of dielectric constant ε_r

If N is the number of atoms/unit volume of the material, μ is the atomic dipole moment, then we have,

Dipole moment/unit volume = $N\mu$ --------------------- (1)

Here the field experienced by the atoms is the internal field E_i. Hence, if α_e is the electronic polarizability of the atoms, we can write the equation for μ as,

$$\mu = \alpha_e E_i \qquad \text{---- (2)}$$

Therefore, dipole moment/unit volume $= N\alpha_e E_i$ --------------------------- (3)

But dipole moment/ unit volume is Polarization P

Therefore $P = N\alpha_e E_i$ -------------------- (4)

Or $E_i = \frac{P}{N\alpha_e}$ -------------------- (5)

But $P = \varepsilon_0(\varepsilon_r - 1)E$

Or $E = \frac{P}{\varepsilon_0(\varepsilon_r - 1)}$ ---------------- (6)

For 3-D lattice, internal field is given by

$$E_i = E + \frac{\gamma P}{\varepsilon_0}$$

Substituting for E_i and E from Eqs(5) and (6) we have

$$\frac{P}{N\alpha_e} = \frac{P}{\varepsilon_0(\varepsilon_r - 1)} + \frac{\gamma P}{\varepsilon_0} \text{ ------------- (7)}$$

Considering the internal field in the material to be Lorentz field, we have $\gamma = 1/3$

$$\frac{1}{N\alpha_e} = \frac{1}{\varepsilon_0(\varepsilon_r - 1)} + \frac{1}{3\varepsilon_0}$$

$$\frac{1}{N\alpha_e} = \frac{3 + (\varepsilon_r - 1)}{3\varepsilon_0(\varepsilon_r - 1)}$$

$$\frac{3\varepsilon_0}{N\alpha_0} = \frac{\varepsilon_r + 2}{\varepsilon_r - 1}$$

By rearranging we have,

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}$$

Numericals:

1. Calculate the probability of an electron occupying an energy level 0.04 eV above the Fermi level at 400 K in a material.

Solution:

Given, T = 400K,

To find f(E) = ?

Above the Fermi Level, $E - E_F = 0.04eV$

We have

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

$$f(E) = \frac{1}{1 + e^{\frac{0.04\times1.6\times10^{-19}}{1.38\times10^{-23}\times400}}}$$

$$= \mathbf{0.238}$$

2. Calculate the Fermi velocity and mean free path for conduction electrons in Al, given that its Fermi energy is 11.63 eV and relaxation time for free electrons is $7.3 \times 10^{-15}s$.

Solution:

Given: $E_F = 11.63\ eV,\ \tau = 7.3 \times 10^{-15}s$

To find; $v_F = ?\ \lambda_F = ?$

We have $E_F = \frac{mv_F^2}{2}$

$$v_F = \sqrt{\frac{2E_F}{m}}$$

$$v_F = \sqrt{\frac{2 \times 11.63 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= \mathbf{2.022 \times 10^6 m/s}$$

3. Find the temperature at which there is 1% probability that a state with energy 0.5 eV above Fermi energy is occupied.

Solution:

Given: $\text{f(E)} = 1\% = \frac{1}{100}$

Above the Fermi Level, $E - E_F = 0.5\ eV$

To find, T =?

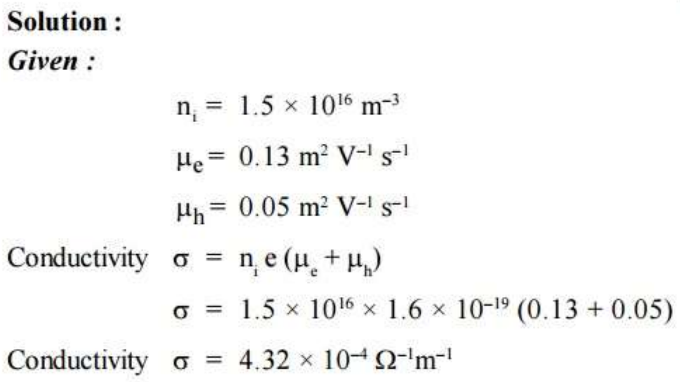
We have

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

$$\frac{1}{100} = \frac{1}{1 + e^{\frac{0.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times T}}}$$

$$\boldsymbol{T = 1261\ K}$$

4. The intrinsic carrier density is $1.5 \times 10^{16}\ \text{m}^{-3}$. If the mobility of electron and hole are 0.13 and 0.05 $\text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$, calculate the conductivity.



Solution :

Given :

$$n_i = 1.5 \times 10^{16}\ \text{m}^{-3}$$

$$\mu_e = 0.13\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$$

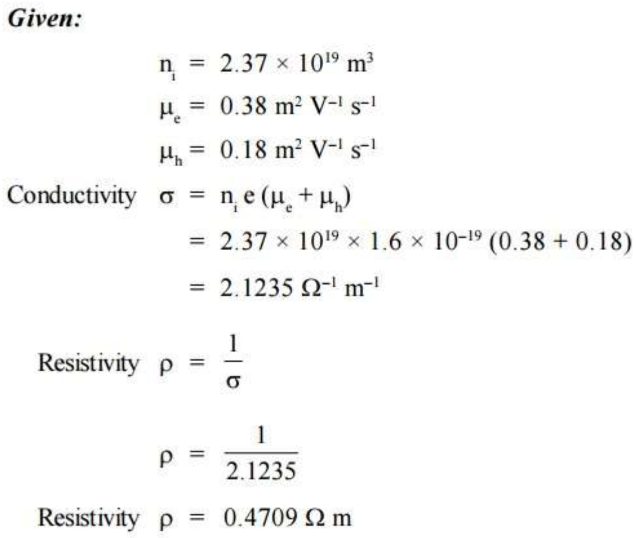
$$\mu_h = 0.05\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$$

$$\text{Conductivity} \quad \sigma = n_i\, e\, (\mu_e + \mu_h)$$

$$\sigma = 1.5 \times 10^{16} \times 1.6 \times 10^{-19}\ (0.13 + 0.05)$$

$$\text{Conductivity} \quad \sigma = 4.32 \times 10^{-4}\ \Omega^{-1}\text{m}^{-1}$$

5. The Intrinsic carrier density at room temperature in Ge is $2.37 \times 10^{19}\ \text{m}^3$ if the electron and hole mobilities are 0.38 and 0.18 $\text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$ respectively, calculate the resistivity.



Given:

$$n_i = 2.37 \times 10^{19}\ \text{m}^3$$

$$\mu_e = 0.38\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$$

$$\mu_h = 0.18\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$$

$$\text{Conductivity} \quad \sigma = n_i\, e\, (\mu_e + \mu_h)$$

$$= 2.37 \times 10^{19} \times 1.6 \times 10^{-19}\ (0.38 + 0.18)$$

$$= 2.1235\ \Omega^{-1}\ \text{m}^{-1}$$

$$\text{Resistivity} \quad \rho = \frac{1}{\sigma}$$

$$\rho = \frac{1}{2.1235}$$

$$\text{Resistivity} \quad \rho = 0.4709\ \Omega\ \text{m}$$

6.The Hall coefficient of certain silicon specimen was found to be -7.35×10^{-5} m^3 C^{-1} from 100 to 400 K. Determine the nature of the semiconductor. If the conductivity was found to be 200Ω^{-1} m^{-1}. Calculate the density and mobility of the charge carrier.

$$\text{Conductivity } \sigma = 200\ \Omega^{-1}\text{m}^{-1}$$

$$\text{Hall co-efficient } R_H = -7.35 \times 10^{-5}\text{m}^3\text{C}^{-1} \qquad \ldots(1)$$

a) Density of electrons

$$n = \frac{-1}{R_H e} \text{ (from equation (1))}$$

$$n = \frac{1}{(7.35 \times 10^{-5} \times 1.609 \times 10^{-19})}$$

$$\text{(i.e)} = 8.455 \times 10^{22}\text{m}^{-3}$$

We know Conductivity

$$\sigma = n\, e\, \mu_e$$

b) Mobility

$$\mu = \frac{\sigma}{ne} = \frac{200}{8.455 \times 10^{22} \times 1.6 \times 10^{-19}}$$

$$= 0.0147$$

$$\text{Mobility } \mu = 0.0147\text{m}^2\text{v}^{-1}\text{s}^{-1}$$

$$\text{Density of electrons } (n) = 8.053 \times 10^{22}\ \text{m}^{-3}$$

$$\text{Mobility } (\mu) = 0.0147\text{m}^2\text{v}^{-1}\text{s}^{-1}$$

7. An N-type semiconductor has hall coefficient = 4.16×10^{-4} m^3 C^{-1}. The conductivity is 108 $^{-1}$ m^{-1}. Calculate its charge carrier density 'n_e'and electron mobility at room temperature**.**

Given:

$$\text{Hall Co-efficient } R_H = 4.16 \times 10^{-4}\ \text{m}^3\ \text{C}^1$$

$$\text{Conductivity } \sigma = 108\ \Omega\text{m}^{-1}$$

1. For 'n' type the charge carriers density $n_e = \frac{-1}{R_H e}$ Here the negative signindicates the field direction alone.

$$n_e = \frac{3\pi}{8}\frac{-1}{R_H e}$$

$$n_e = \left[\frac{3 \times 3.14}{8}\right]\left[\frac{1}{1.6 \times 10^{-19} \times 4.6 \times 10^{-4}}\right]$$

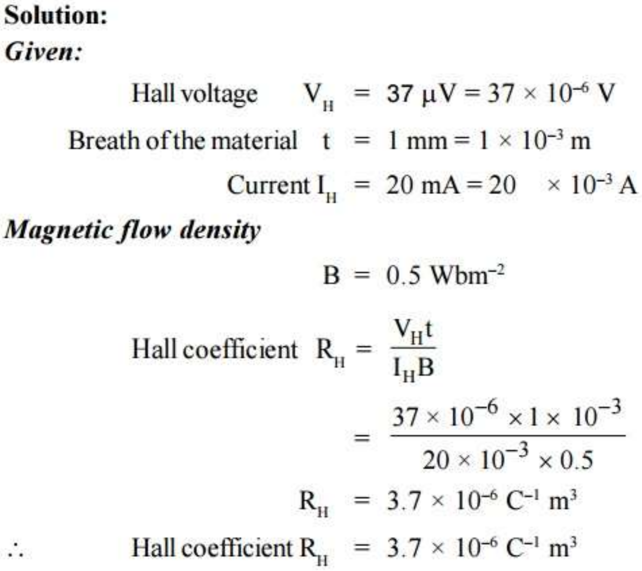
$$n_e = 1.7690 \times 10^{22}\text{m}^{-3}$$

2. Electron mobility $\mu_e = \frac{\sigma_e}{n_e e}$

$$= \frac{108}{\left(1.7690 \times 10^{22} \times 1.6 \times 10^{-19}\right)}$$

$$\mu_e = 0.0381\ \text{m}^2\text{v}^{-1}\text{s}^{-1}$$

8. A semiconducting crystal with 12 mm long, 5 mm wide and 1 mm thick has a magnetic density of 0.5 Wbm^{-2} applied from front to back perpendicular to largest faces. When a current of 20 mA flows length wise through the specimen, the voltage measured across its width is found to be $37\mu V$. What is the Hall coefficient of this semiconductor?



Solution:

Given:

Hall voltage $V_H = 37\ \mu V = 37 \times 10^{-6}$ V

Breath of the material $t = 1$ mm $= 1 \times 10^{-3}$ m

Current $I_H = 20$ mA $= 20 \times 10^{-3}$ A

Magnetic flow density

$B = 0.5$ Wbm^{-2}

$$\text{Hall coefficient } R_H = \frac{V_H t}{I_H B}$$

$$= \frac{37 \times 10^{-6} \times 1 \times 10^{-3}}{20 \times 10^{-3} \times 0.5}$$

$$R_H = 3.7 \times 10^{-6}\ C^{-1}\ m^3$$

$\therefore$ Hall coefficient $R_H = 3.7 \times 10^{-6}\ C^{-1}\ m^3$

9. The Electronic polarizability of Krypton gas is $3.54 \times 10^{-40} Fm^2$. If the gas contains $2.7 \times \frac{10^{25} atoms}{m^3}$ at NTP calculate the dielectric constant of Krypton.

Solution:

Given:

Electronic polarizability $\alpha_e = 3.54 \times 10^{-40} Fm^2$

Number of atoms per unit volume, $N = 2.7 \times 10^{25} atoms/m^3$

Dielectric constant, $\varepsilon_r = K$=?

We have

$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$$

Substituting the given values,

$$3.54 \times 10^{-40} = \frac{8.854 \times 10^{-12}(\varepsilon_r - 1)}{2.7 \times 10^{25}}$$

Simplifying, we get

$$\boldsymbol{\varepsilon_r = 1.00108}$$

10. An elemental solid dielectric material has polarizability $7 \times 10^{-40} Fm^2$. Assuming the internal field to be Lorentz field, calculate the dielectric constant for the material if the material has $3 \times 10^{28} atoms/m^3$.

Solution:

Electronic polarizability $\alpha_e = 7 \times 10^{-40} Fm^2$

Number of atoms per unit volume, $N = 3 \times 10^{28} atoms/m^3$

Dielectric constant, ε_r =?

Since, the internal field is Lorentz field we can apply Clausius - Mossotti equation given by

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}$$

$$\frac{3\times10^{28}\times7\times10^{-40}}{3\times8.854\times10^{-12}} = \frac{\varepsilon_r-1}{\varepsilon_r+2}$$

$$\boldsymbol{\varepsilon_r = 12.33}$$

11. Find the polarization produced in a dielectric medium of relative permittivity 15 in the presence of an electric field of 500 V/m.

Solution:

Given:

Relative permittivity = Dielectric constant, ε_r =15

Electric field, E = 500 V/m

Polarization, P = ?

We have,

$$P = \varepsilon_0(\varepsilon_r - 1)E$$

Substituting we get,

$$P = 8.854 \times 10^{-12} \times (15 - 1)500$$

$$\mathbf{P= 6.1978 \times 10^{-8} C/m^{-2}}$$