MODULE 2
DC Circuits and Single-phase AC Circuits

Syllabus:
DC Circuits: Ohms Law, Voltage division & Current division in DC circuits; Analysis of series, Parallel and series-parallel circuits excited by independent voltage sources; Power and Energy; Kirchhoff's Laws.

Single-phase AC Circuits: Generation of sinusoidal voltage, Frequency of generated voltage, Average value, R.M.S value, Form factor and Peak factor; Voltage, current and power analysis of basic R, L, C, RL, RC and RLC circuits with phasor Diagrams. Concept of Real power, Reactive power, Apparent power and Power factor.

DC Circuits:

Ohm's Law

This law applies to electric conduction through good conductors. This law gives relationship between the voltage or potential difference (V), the current (I) and the resistance (R) of a circuit.

Statement of Ohm's Law:

"The voltage across a conductor is directly proportional to the current flowing through it, provided all physical conditions and temperature, remain constant". That is

$$V \propto I \text{ or } V = RI$$

Where R = constant of proportionality = resistance of the conductor. Its unit is ohm(Ω).

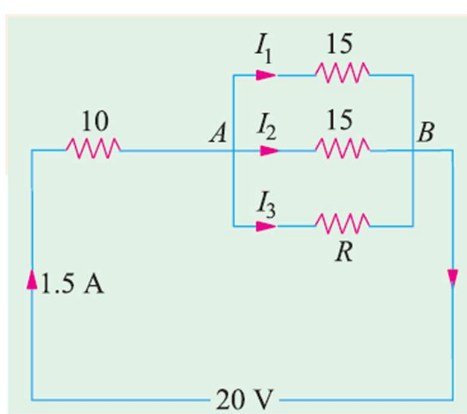
$$\text{Hence, } \frac{V}{I} = R$$

Limitations of Ohm's Law

The limitations of the Ohm's law are,
1. It is not applicable to the nonlinear devices such as diodes, Zener diodes, voltage regulators etc.
2. It does not hold good for non-metallic conductors such as silicon carbide.

Problems:

1. A resistance of 10 Ω is connected in series with two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied?



Drop across 10-Ω resistor = 1.5 × 10 = 15 V
Drop across parallel combination, VAB = 20 − 15 = 5 V
Hence, voltage across each parallel resistance is 5 V.
I1 = 5/15 = 1/3 A, I2 = 5/15 = 1/3 A
I3 = 1.5 − (1/3 + 1/3) = 5/6 A
∴ I3 R = 5 or (5/6) R = 5 or R = 6 Ω

2. A Battery has an emf of 12.8 volts and supplies a current of 3.24 A. What is the resistance of the circuit?
Sol: Circuit Resistance, R = V/I = 12.8/3.24 = 4 Ω.

Series Circuit

A series circuit is one in which several resistances are connected one after the other. Such connection is also called end to end connection or cascade connection. There is only one path for the flow of current.

Consider the resistances shown in the Fig. 2.1

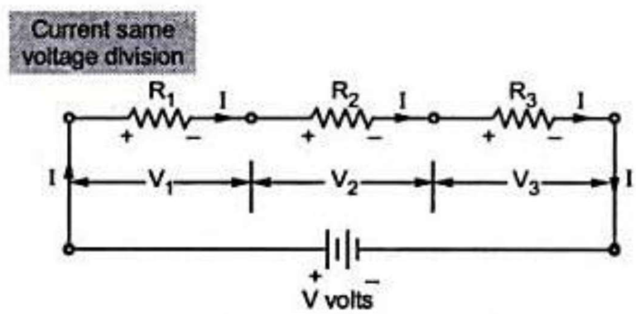


Fig 2.1 Series circuit

The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. Ex: the chain of small lights, used for the decoration is good example of series combination.
Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively.

Then, $V = V_1 + V_2 + V_3$

Where $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$ (from Ohm's law)
Current through all the resistors is the same, that is I
Therefore, $V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$ ----- *Eqn 1*

Applying Ohm's law to the overall circuit

$V = I R_{eq}$ ----- *Eqn 2*

Where Req = equivalent resistance of the circuit.
By comparing *Eqn 1* and *Eqn 2*

$$R_{eq} = R_1 + R_2 + R_3$$

The total or equivalent resistance of series circuit is the arithmetic sum of resistance connected in series.

Characteristics of Series Circuits

- The same current flows through each resistance.
- The equivalent resistance is equal to the sum of the individual resistances
- The voltage drop across each resistor will be different.
- The supply voltage V is the sum of the individual voltage drops across the resistances.
  $V = V_1+V_2+\ldots\ldots\ldots +V_n$

Parallel Circuit

The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

Consider a parallel circuit shown in the Fig. 2.2 below

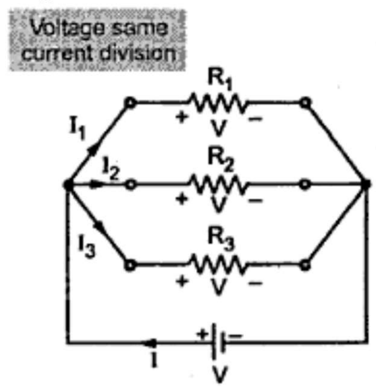


Fig 2.2 Parallel circuit

In the circuit shown above, the three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V'.

Let total current drawn is say 'I' as shown.

There are 3 paths for this current (through R_1, R_2 and R_3).

I_1 = Current through R_1

I_2= Current through R_2

I_3= Current through R_3

Voltage drop across each resistor is the same and equals to the supply voltage V.

By applying Ohm's law to each resistance we get,

$$I_1=\frac{V}{R1} \qquad I_2=\frac{V}{R2} \qquad I_3=\frac{V}{R3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R1}+\frac{V}{R2}+\frac{V}{R3}$$

$$I = V\left(\frac{1}{R1}+\frac{1}{R2}+\frac{1}{R3}\right) \text{ ------ } Eqn\ 1$$

For the overall circuit if ohm`s law is applied,

$$V = I\ Req$$

Where Req = equivalent resistance of the circuit.

So, $\boldsymbol{I = \frac{V}{Req}}$ ***----- Eqn 2***

By comparing *Eqn 1*and *Eqn 2*

$$\frac{1}{\text{Req}} = \frac{1}{\text{R1}} + \frac{1}{\text{R2}} + \frac{1}{\text{R3}}$$

In general, if **n** resistors are connected in parallel,

$$\frac{1}{\text{Req}} = \frac{1}{\text{R1}} + \frac{1}{\text{R2}} + \frac{1}{\text{R3}} + \ldots\ldots + \frac{1}{\text{Rn}}$$

Now if **n=2**, two resistance are in parallel, then,

$$\frac{1}{\text{Req}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{\text{R1R2}}{\text{R1} + \text{R2}}$$

Characteristics of Parallel Circuits

- The potential difference across each resistor is the same.
- The current through each resistor is different.
- The total current is always sum of all the branch currents. $I = I_1 + I_2 + I_3 + \ldots + I_n$.
- Resistance of the circuit is $\frac{1}{\text{Req}} = \frac{1}{\text{R1}} + \frac{1}{\text{R2}} + \frac{1}{\text{R3}} + \ldots\ldots + \frac{1}{\text{Rn}}$.

Voltage Division in Series Circuit of Resistors

Consider a series circuit of two resistors R_1 and R_2 connected to source of V volts (Fig. 2.3). As two resistors are connected in series, the current flowing through both the resistors is same, i.e. I.

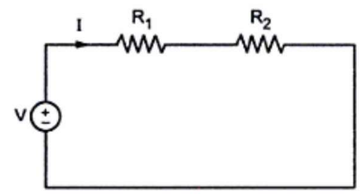


Fig 2.3 Voltage divider circuit

Then applying KVL, we get, $\mathbf{V = IR_1 + IR_2}$

$$I = \frac{V}{(R1 + R2)}$$

Let V_{R1} = Voltage drop across R1
V_{R2} = Voltage drop across R2
Therefore, $\mathbf{V_{R1} = IR_1}$

$$V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V$$

Similarly, $\mathbf{V_{R2} = IR_2}$

$$V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V$$

So, in general, voltage drop across any resistors or combination of resistors in a series circuit is equal to the ratio of that resistance value, to the total resistance multiplied by the source voltage.

Current Division in Parallel Circuit of Resistors

Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts (Fig. 2.4). Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

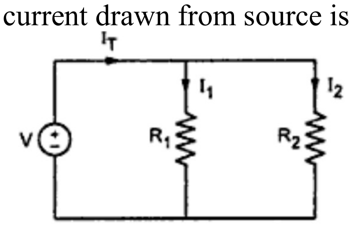


Fig 2.4 Current division circuit

By KCL, we get, $I_T = I_1 + I_2$

But $I_1 = \frac{V_1}{R_1}$ & $I_2 = \frac{V_2}{R_2}$

and $V = I_1R_1 = I_2R_2$

$$\text{Hence, } \mathbf{I_T = I_2\left(\frac{R_2}{R_1}\right)}$$

Substituting the value of I_1 in I_T

$$\mathbf{I_T = I_2\left(\frac{R2}{R1}\right) + I_2 = I_2\left[\left(\frac{R_2}{R_1}\right) + 1\right] = I_2\left(\frac{R_2+R_1}{R_1}\right)}$$

$$\mathbf{I_2 = I_T\left(\frac{R_1}{R_1 + R_2}\right)}$$

Now, $I_1 = I_T - I_2 = I_T - I_T\left(\frac{R_1}{R_1 + R_2}\right)$

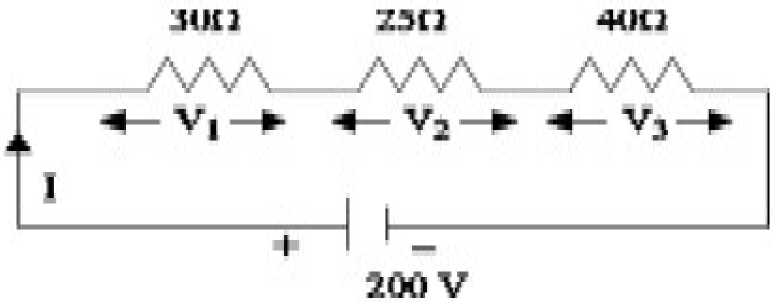
$$I_1 = I_T\left(\frac{R_1 + R_2 - R_1}{R_1 + R_2}\right)$$

$$I_1 = I_T\left(\frac{R_2}{R_1 + R_2}\right)$$

In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.

Problem:

1. Three resistors 30 Ω, 25 Ω, 45 Ω are connected in series across 200V. Calculate (i) Total resistance (ii) Current (iii) Potential difference across each element.



(i) Total Resistance (R_T)

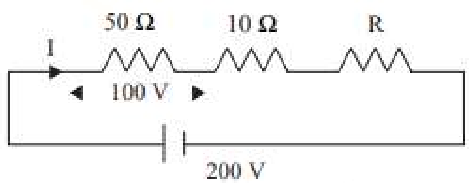
$$R_T = R_1 + R_2 + R_3$$
$$R_T = 30 + 25 + 45 = 100\ \Omega$$

(ii) Current, $I = \frac{V}{R_T} = \frac{200}{100} = 2\ A$

(iii) Potential difference across each element,

$$V_1 = IR_1 = 2 * 30 = 60\ V$$
$$V_2 = IR_2 = 2 * 25 = 50\ V$$
$$V_3 = IR_3 = 2 * 45 = 90\ V$$

2. Find the value of 'R' in the circuit diagram, given below.



We know that, $V_1 = IR_1$

$$I = V_1 / R_1 = 100/50 = 2\ A$$

Similarly, $V_2 = IR_2 = 2 * 10 = 20\ V$

Total voltage drop, $V = V_1 + V_2 + V_3$

$$V_3 = V - (V_1 + V_2) = 200 - (100 + 20)$$
$$V_3 = 80\ V$$
$$V_3 = IR_3,\ R_3 = V_3 / I = 80/2 = 40\ \Omega$$
$$\therefore R_3 = 40\ \Omega$$

Power (P): Power in a DC circuit is the rate at which electrical energy is transferred or converted within the circuit. It is the amount of work done or energy consumed (or produced) per unit of time. Mathematically, power in a DC circuit is calculated using Ohm's Law:

P=VI

Where:

P is the power in watts (W)
V is the voltage in volts (V)
I is the current in amperes (A)

This equation demonstrates that power is directly proportional to both voltage and current in a DC circuit.

Energy (E): Energy in a DC circuit is the capacity to do work or the total amount of electrical potential and kinetic energy stored or used in the circuit. It represents the cumulative power consumed or produced over a specific period. The energy consumed or produced in a DC circuit is calculated using the following equation:

$$E=Pt$$

Where:
E is the energy in watt-hours (Wh) or joules (J)
P is the power in watts (W)
t is the time in hours (h) for watt-hours or seconds (s) for joules.

This equation illustrates that energy is the product of power and time.
In summary, power represents the rate of energy transfer or conversion in a DC circuit, while energy is the total amount of work done or consumed over a given time.

Kirchhoff's Current Law or Point Law (KCL)

It states as follows:

"The algebraic sum of all currents entering and leaving a node must equal to zero"
In other words, the total current leaving a junction is equal to the total current entering that junction.
Consider the case of a few conductors meeting at a point A as in Fig. 2.5 below

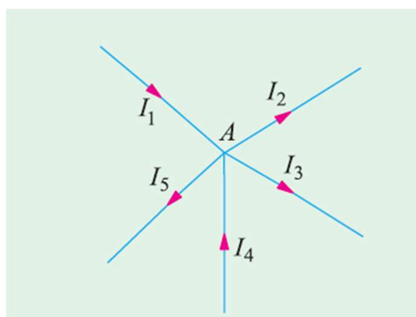


Fig. 2.5 Illustration of KCL

Some currents are moving towards point A, whereas some currents are moving away from point A. Assuming the incoming currents to be positive and the outgoing currents negative, we have

$$I_1 - I_2 - I_3 + I_4 - I_5 = 0$$

$$\text{or } I_1 + I_4 - I_2 - I_3 - I_5 = 0 \text{ or } I_1 + I_4 = I_2 + I_3 + I_5$$

or sum of incoming currents = sum of outgoing currents

Kirchhoff's Voltage Law or Mesh Law (KVL)

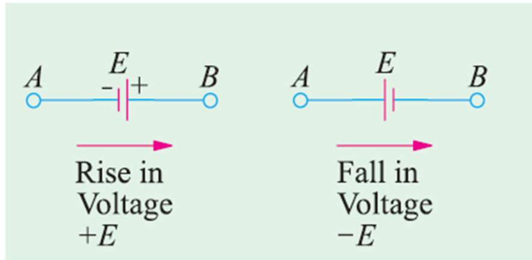
It states as follows:

"The algebraic sum of all voltage drops around any closed loop is zero."

In other words, $\Sigma\, IR + \Sigma\, e.m.f. = 0$ ...round a mesh

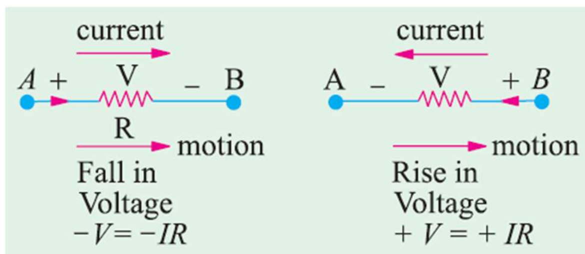
It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

Sign conventions to be followed while applying KVL:



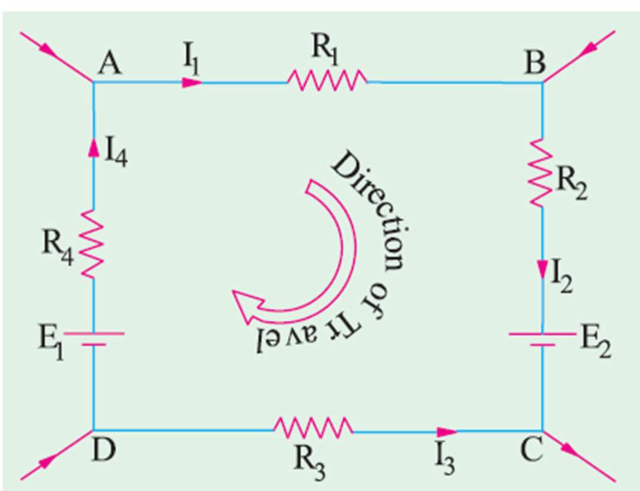
If we move from negative terminal to the positive terminal of the battery, there is a rise in potential and it is represented by a + sign

If we move from positive terminal to the negative terminal of the battery, there is a fall in potential and it is represented by a – sign



If we go through a resistor in the same direction as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken negative.

However, if we go in a direction opposite to that of the current, then there is a rise in voltage. Hence, this voltage rise should be given a positive sign.



I_1R_1 is − ve (fall in potential)

I_2R_2 is − ve (fall in potential)

I_3R_3 is + ve (rise in potential)

I_4R_4 is − ve (fall in potential)

E_2 is − ve (fall in potential)

E_1 is + ve (rise in potential)

Using Kirchhoff's voltage law, we get

$-I_1R_1 - I_2R_2 + I_3R_3 - I_4R_4 - E_2 + E_1 = 0$

or $\mathbf{I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2}$

EXERCISE PROBLEMS:

1. Determine the voltage across each resistor in the following circuit shown in Fig. 1 using voltage division rule

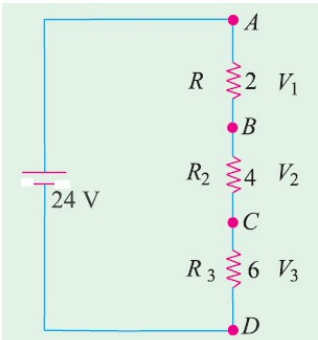


Fig. 1

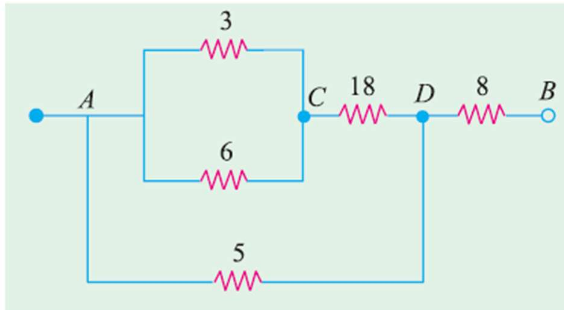
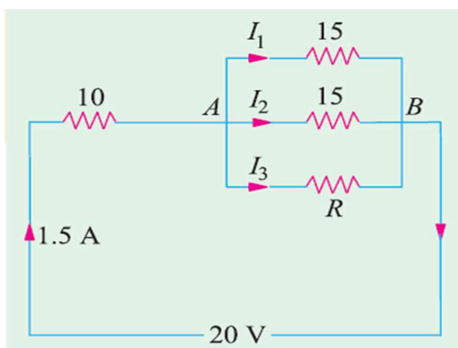


Fig. 2

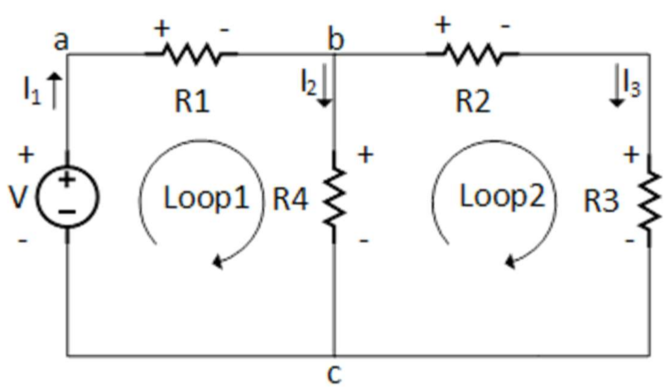
2. Calculate the effective resistance and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B for the circuit shown in Fig. 2

3. A resistance of 10 Ω is connected in series with two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied?



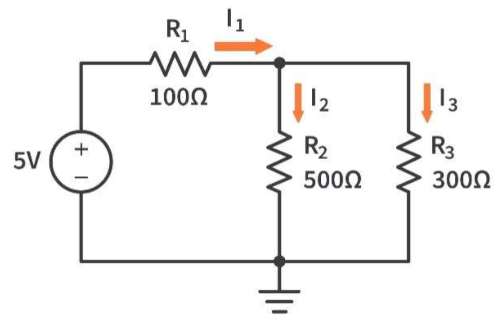
4. Find the current and voltage of each element of the circuit for the following given circuit parameter using Kirchhoff's voltage law.

$R_1 = 5\Omega R_2 = 10\Omega R_3 = 5\Omega R_4 = 10\Omega V = 20$Volts



(Ans: I1=1.81A, I2=1.09A, I3=0.72A, VR1=9.05V, VR2=7.2V, VR3=3.6V, VR4=10.9V)

5. Determine currents I1, I2 and I3 in the circuit given below



(Ans: I1=17.4mA, I2=6.52mA, I3=10.87mA)

Single-phase AC Circuits

Today, the vast majority of electrical power is generated, distributed, and consumed in the form of A.C power. The term ‘A.C’ means ‘Alternating Current’ which reverses its magnitude and direction periodically, whereas, Direct Current (D.C) has constant magnitude with respect to time. Fig. 2.6 shows the graph of such current with respect to time.

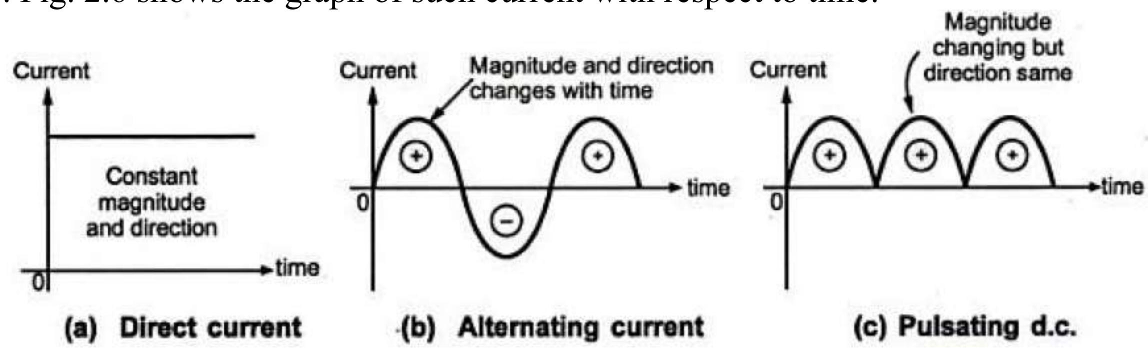


Fig 2.6

Such change in magnitude and direction is measured in terms of cycle. Each cycle of a.c. consists of two half cycles namely positive cycle and negative cycle. Current increases in magnitude, in one particular direction, attains maximum and start decreasing, passing through zero it increases in opposite direction and behaves similarly.

In practice some waveform are available in which magnitude changes but its direction remains same as positive or negative. Such waveform is called pulsating d.c. The waveform obtained as output of full wave rectifier is an example of pulsating d.c.

$$\mathbf{e = E_m \sin\omega t = E_m \sin(2\pi f)t}$$

where B l v = E_m = maximum value of the emf induced. Similar equation can be written for current also, i = I_m sin(2πft) or $I_m$sinθ.

STANDARD TERMINOLOGIES RELATED TO ALTERNATING QUANTITY

The equations of AC voltage and current are as given below:

$$\mathbf{v = V_m \sin\omega t = V_m \sin(2\pi f)t}$$

where V_m = maximum value of the AC voltage.

Similarly, $\mathbf{i = I_m \sin(2\pi ft) = I_m \sin\omega t}$

The sinusoidally varying alternating emf is as shown in figure 2.7.

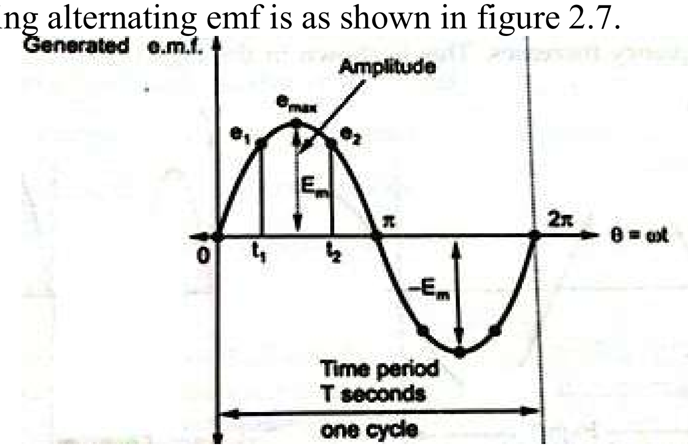


Fig 2.7 Waveform of an alternating e.m.f, e = E_m sinω t

a. Instantaneous value

The value of an alternating quantity at a particular instant is known as **instantaneous value**. For example, e_1 and e_2 are the instantaneous value of an alternating emf at the instant t_1 and t_2 respectively.

b. Waveform

The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

c. Cycle

Each repetition of a set of positive and negative instantaneous values of an alternating quantity is called a **cycle**.

d. Time Period (T)

Time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by T seconds. After every T seconds, the cycle of an alternating quantity repeats.

e. Frequency (f)

The number of cycles completed by an alternating quantity per second is known as its frequency. It is denoted by f and is measured in cycles/second which is known as Hertz, denoted as Hz. $\mathbf{f = \frac{1}{T}}$ **Hz**.

f. Amplitude or Peak value

The maximum value attained by an alternating quantity during positive or negative half cycle is called its amplitude or peak value. It is denoted as E_m or I_m.

g. Angular Frequency (ω)

It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as (2π * cycles/sec) and its unit is radians/sec.

$$\omega = 2\pi f \text{ radians/sec.}$$

h. Effective value or R.M.S value (I_{rms} or I)

This is also called as the *root mean square value* or *r.m.s value* in short. It is defined on the basis of amount of heat produced.

*The **effective value or r.m.s. value** of an alternating current is equal to that steady current, which produces the same amount of heat as produced by the alternating current, when passed through the same resistance for the same time.*

$$\therefore \mathbf{I} = \frac{\mathbf{I_m}}{\sqrt{2}} = \mathbf{0.707 I_m}.$$

i. Average value (I_{av})

This is defined on the basis of the amount of charge transferred.

*The **average value** of an alternating current is equal to that steady current, which transfers the same amount of charge, as transferred by the alternating current across the same circuit and in the same time.*

$$\therefore \; I_{av} = \frac{2I_m}{\pi} = 0.637 I_m.$$

j. Form factor (K_f)

The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value.

$$K_f = \frac{\text{r.m.s value}}{\text{average value}}$$

The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\, I_m}{0.637\, I_m} = 1.11.$$

k. Crest or Peak factor(K_P)

The peak value of an alternating quantity is defined as the ratio of maximum value to the r.m.s. value.

$$K_P = \frac{\text{maximum value}}{\text{r.m.s. value}}$$

The peak factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_P = \frac{I_m}{0.707\, I_m} = 1.414.$$

l. Phase and Phase Difference of an alternating quantity

Phase of an alternating quantity is the angle through which the rotating vector representing the alternating quantity has rotated through from the reference axis as shown in fig. 2.8.

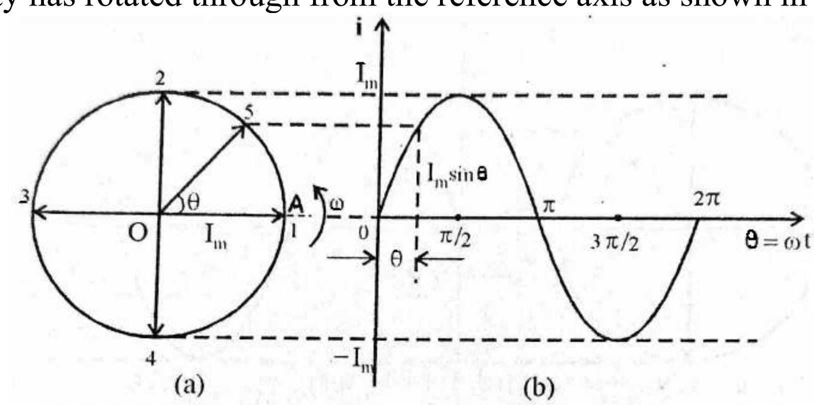


Fig. 2.8

Phase difference between the two alternating quantities is the angle difference between the two rotating vectors representing the two alternating quantities.

<u>Case 1</u>**: Current *In-Phase* with the Voltage**

The rotating vector OA represents the alternating voltage and OB represents the alternating current. Both of them rotate together with an angular velocity $\boldsymbol{\omega}$ and phase difference is zero. Two quantities are said to be in phase with each other when their corresponding values occur at the same time as shown in fig. 2.9

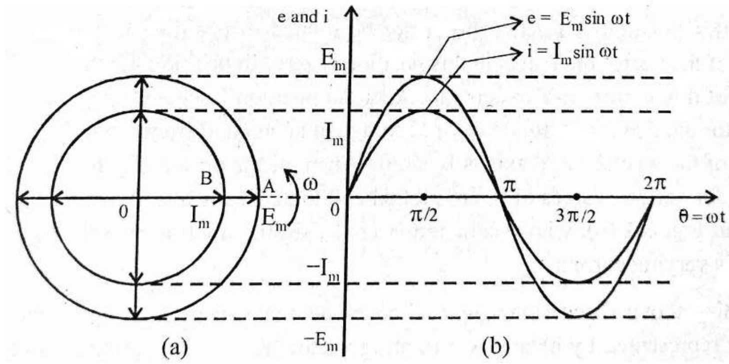


Fig. 2.9 Current *in-phase* with voltage

The equations for voltage and current are,

$$\mathbf{e = E_m \sin\omega t} \text{ and } \mathbf{i = I_m \sin\omega t}.$$

Case 2: Current *Lagging* the Voltage:

Vectors OA and OB rotate with an angle difference of ϕ, the current vector always lags the voltage vector by an angle ϕ as shown in fig. 2.10

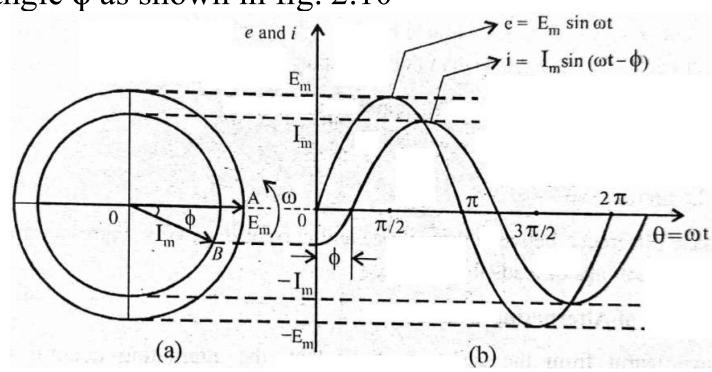


Fig. 2.10 Current *lagging* the voltage

The equations for voltage and current are,

$$\mathbf{e = E_m \sin\omega t} \text{ and } \mathbf{I = I_m \sin(\omega t - \phi)}$$

Case 3: Current *Leading* the Voltage

Vectors OA and OB rotate with an angle difference of ϕ, current vector leads the voltage vector by an angle ϕ as shown in fig. 2.11

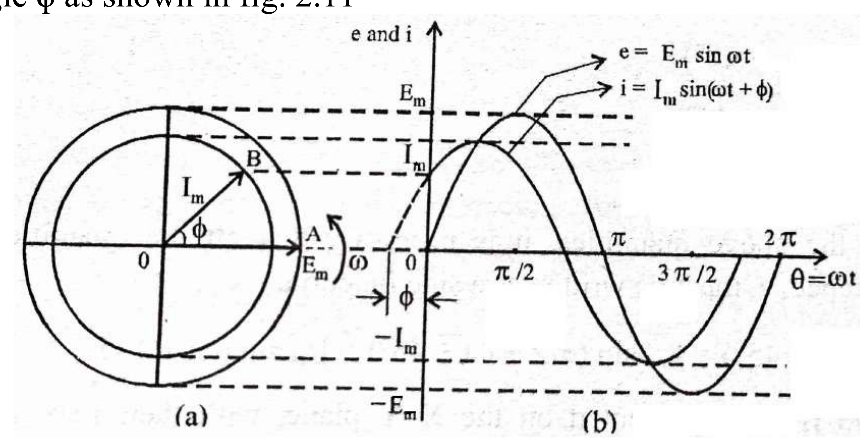


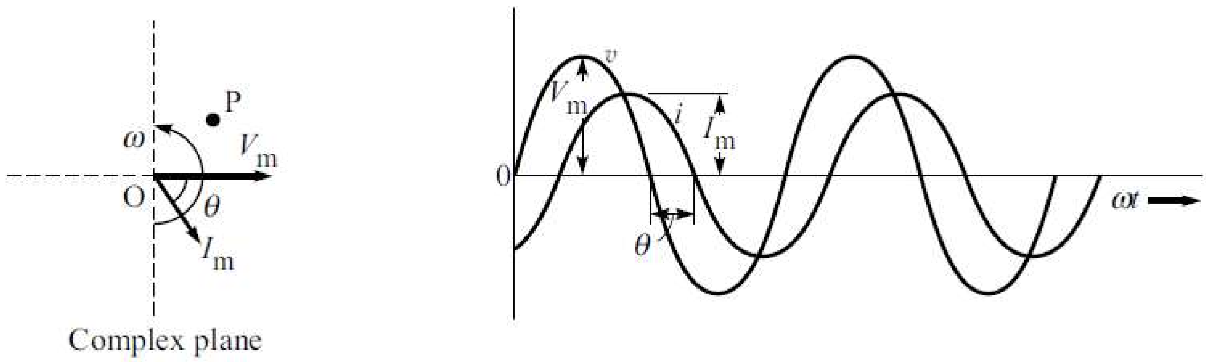
Fig. 2.11 Current *leading* the voltage

The equations for current and voltage are,

$$\mathbf{e = E_m \sin\omega t} \text{ and } \mathbf{i = I_m \sin(\omega t+\phi)}.$$

m. Phasor Diagram

It is a diagram containing the phasors of inter-related sinusoidal voltages and currents,with their phase differences indicated. Thus, Fig. 2.12a is a phasor diagram showing the amplitudes and phasor relationship of voltage and current. This diagram and the time-function waveforms shown in Fig. 2.12b convey the same information about the voltage and current. Obviously, drawing the phasor diagram needs much less effort than drawing the time-varying waveforms. Therefore, electrical engineers prefer to represent inter-related sinusoidal quantities by phasors rather than by time waveforms.



(*a*) Rotating voltage and current phasors.

(*b*) Waveforms produced.

Fig. 2.12

Useful Formulae	
$-\sin\omega t = \sin(\omega t \pm 180°)$;	$-\cos\omega t = \cos(\omega t \pm 180°)$
$\pm\sin\omega t = \cos(\omega t \pm 90°)$;	$\pm\cos\omega t = \sin(\omega t \pm 90°)$

AVERAGE VALUE OF A SINE WAVE

This is defined on the basis of the amount of charge transferred.

The ***average value*** *of an alternating current is equal to that steady current, which transfers the same amount of charge, as transferred by the alternating current across the same circuit and in the same time.*

By definition, average value is equal to the sum of all the values divided by the number of values.

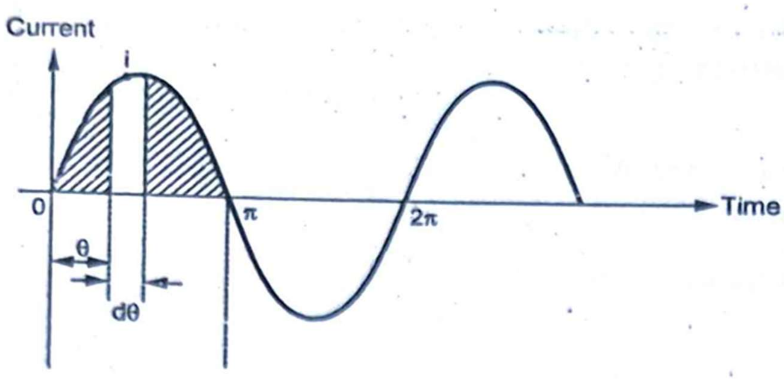


Fig. 2.13 Average value of a sinusoidal waveform

$$V_{av} = \frac{\text{Area under full cycle}}{\text{Length of one cycle}} = \frac{\int_0^{2\pi} v\,d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} v\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} v\,d(\omega t)$$

$$V_{av} = \frac{1}{T}\int_0^{T} v\,dt$$

In a sinusoidal waveform, both positive and negative cycles will have the same area and the algebraic sum these two areas become zero. Hence, in a sinusoidal waveform, average value is calculated over one half cycle only.

$$I_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}$$

$$I_{av} = \frac{\int_0^{\pi} i\,d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i\;d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[-\cos\pi + \cos 0\right] = \frac{I_m}{\pi}[\,2\,]$$

$$= \frac{2\,I_m}{\pi} = 0.637\ I_m$$

Hence, the average values of sinusoidal current and voltages are gives as follows:

$$I_{av} = 0.637\ I_m \quad \text{and} \quad V_{av} = 0.637\ V_m$$

RMS VALUE OF A SINE WAVE

This is also called as the *root mean square value* or *r.m.s value* in short. It is defined on the basis of amount of heat produced.

*The **effective value or r.m.s. value** of an alternating current is equal to that steady current, which produces the same amount of heat as produced by the alternating current, when passed through the same resistance for the same time.*

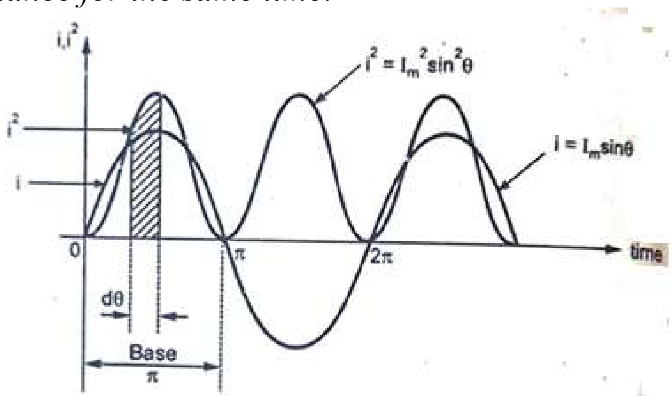


Fig. 2.14 Current i and Square of current i^2 waveforms

Step 1: Find the average value of square of sinusoidal current over a half cycle:

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^\pi i^2 \, d\theta}{\pi} = \frac{1}{\pi}\int_0^\pi i^2 \, d\theta = \frac{1}{\pi}\int_0^\pi I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^\pi \left[\frac{1-\cos 2\theta}{2}\right] d\theta = \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi = \frac{I_m^2}{2\pi}[\pi] = \frac{I_m^2}{2}$$

Step 2: Find the RMS value of above result

$$I_{r.m.s.} = \sqrt{\text{mean or average of square of current}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I_{r.m.s.} = 0.707\, I_m}$$

SINGLE-PHASE CIRCUITS

The resistance, inductance and capacitance are three basic elements of any electrical network. In order to analyze any electric circuit, it is necessary to understand the following three cases,

a) A.C. through pure resistive circuit.
b) A.C. through pure inductive circuit.
c) A.C. through pure capacitive circuit.

In each case, it is assumed that a purely sinusoidal alternating voltage given by the equation $v = V_m \sin \omega t$ is applied to the circuit. The equation of the current, power and phase shift is developed in each case. The voltage applied having zero phase angle is assumed reference while plotting the phasor diagram in each case.

A.C. THROUGH PURE RESISTANCE

Consider a simple circuit consisting of a pure resistance 'R' Ohm connected across a voltage
$v = V_m \sin\omega t$.

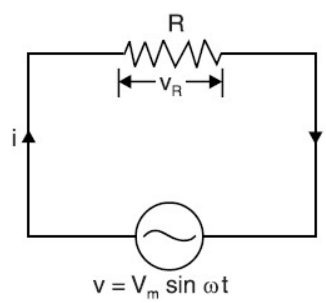


Fig 2.13 Pure resistive circuit

According to Ohm's law, we can find the equation for the current **i** as

$$i = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$$

$$i = \left(\frac{V_m}{R}\right)\sin(\omega t)$$

This is the equation giving instantaneous value of the current. Comparing this with standard equation,

$$i = I_m \sin(\omega t + \phi)$$
$$I_m = \frac{V_m}{R} \text{ and } \phi = 0$$

So the maximum value of alternating current, **i** is $\mathbf{I_m = \frac{V_m}{R}}$ and $\boldsymbol{\phi = 0}$ it indicates that it is in phase with the applied voltage. i.e, there is no phase difference between the two.

The waveform of voltage and current and phasor diagram is shown in Fig 2.14.

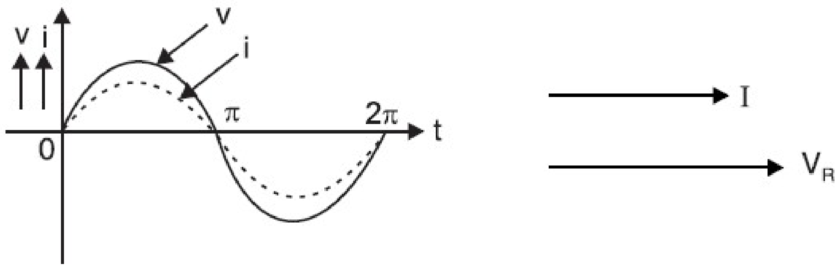


Fig 2.14: Voltage, Current waveforms and phasor diagram

Power in pure resistive circuit

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$\mathbf{P} = \mathbf{v} \times \mathbf{i} = \mathrm{V_m}\sin(\omega t) \times I_\mathrm{m}\sin\omega t = V_\mathrm{m}\mathrm{I_m}\sin^2(\omega t)$$

$$= \frac{V_\mathrm{m}\mathrm{I_m}}{2}(1 - \cos 2\omega t)$$

$$\mathrm{P} = \frac{\mathrm{V_m I_m}}{2} - \frac{\mathrm{V_m I_m}}{2}\cos(2\omega \mathrm{t})$$

From the above equation, it is clear that the instantaneous power consist of two components

a) Constant power component $\frac{\mathbf{V_m I_m}}{\mathbf{2}}$.

b) Fluctuating component $\frac{\mathbf{V_m I_m}}{\mathbf{2}}\mathbf{cos}(\mathbf{2}\boldsymbol{\omega}\mathbf{t})$ having frequency double the frequency of applied voltage.

Now, the average value of the fluctuating component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{\mathbf{V_m I_m}}{\mathbf{2}}$

$$P_\mathrm{av} = P = \frac{\mathrm{V_m I_m}}{2} = \frac{\mathrm{V_m}}{\sqrt{2}} \cdot \frac{\mathrm{I_m}}{\sqrt{2}}$$

$$\mathrm{P} = \mathrm{V_{rms}} \times \mathrm{I_{rms}}\ \text{ Watts}$$

$$\mathrm{P} = \mathrm{V} \times \mathrm{I}\text{Watts } = \mathrm{I^2 R}\ \text{ Watts.}$$

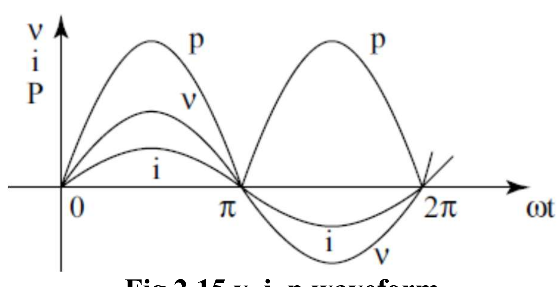


Fig 2.15 v, i, p waveform

A.C. THROUGH PURE INDUCTANCE

Consider a simple circuit consisting of a pure inductance of **L** Henry, connected across a voltage given by the equation, $v = V_m \sin\omega t$.

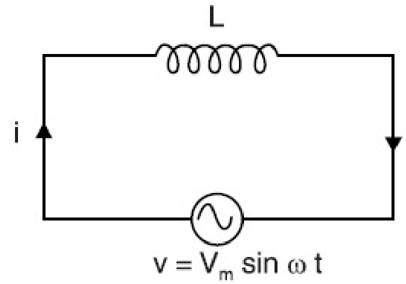


Fig 2.16 Pure Inductive circuit

Pure inductance has zero internal resistance. The coil has pure inductance of **L** Henries. When alternating quantity **i** flows through inductance '**L**', it sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self-inductance, emf gets induced in the coil. This emf opposes the applied voltage.

The self-induced emf in the coil is given by

$$\mathbf{e} = -\mathbf{L}\frac{\mathrm{di}}{\mathrm{d}t}$$

At all instant, the applied voltage **v** is equal and opposite to the self-induced emf

$$\mathbf{v} = -\mathrm{e} = -\left(-\mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}}\right)$$

$$\mathbf{v} = \mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}} \quad \text{i.e,} \quad \mathrm{V_m}\sin\omega\mathrm{t} = \mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}}$$

$$\mathrm{di} = \frac{\mathrm{V_m}}{\mathrm{L}}\sin\omega t\mathrm{dt}$$

$$\mathrm{i} = \int \mathrm{di} = \int \frac{\mathrm{V_m}}{\mathrm{L}}\sin\omega\mathrm{tdt} = \frac{\mathrm{V_m}}{\mathrm{L}}\left(\frac{-\cos\omega\mathrm{t}}{\omega}\right)$$

$$= -\frac{\mathrm{V_m}}{\omega\mathrm{L}}\sin\left(\frac{\pi}{2} - \omega t\right) \qquad \because \cos\omega\mathrm{t} = \sin\left(\frac{\pi}{2} - \omega\mathrm{t}\right)$$

$$\mathrm{i} = \frac{\mathrm{V_m}}{\omega\mathrm{L}}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad \because \sin\left(\frac{\pi}{2} - \omega\mathrm{t}\right) = -\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\mathbf{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

where $\mathbf{I_m} = \frac{v_m}{\omega L} = \frac{v_m}{X_L}$

where $X_L = \omega L = 2\pi f L\ \ \Omega.$

The term, X_L is called **Inductive Reactance** and is measured in **ohms.**

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $\frac{-\pi}{2}$ radians i.e, **-90°**. This means that **the current lags voltage applied by 90°.**
Figure 2.17 shows the waveform and the corresponding phasor diagram.

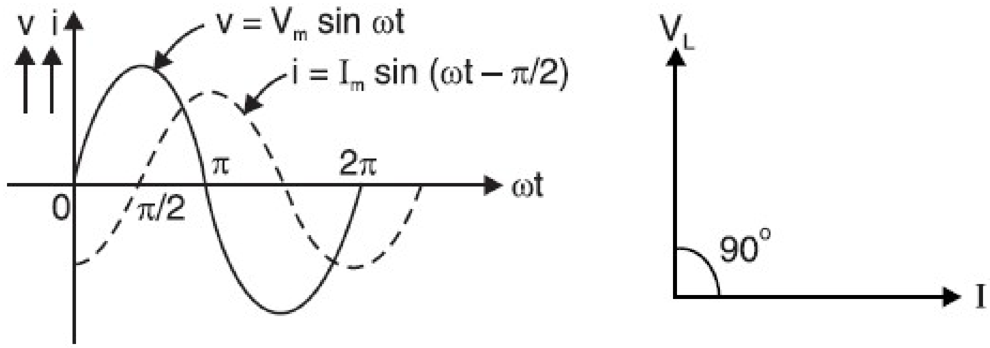


Fig 2.17: Voltage and Current Waveform and Corresponding phasor diagram

Power in pure inductive circuit

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin\omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin(\omega t)\cos(\omega t) \qquad \because \sin\left(\omega t - \frac{\pi}{2}\right) = -\cos\omega t$$

$$P = -\frac{V_m I_m}{2}\sin(2\omega t) \qquad \because 2\sin\omega t\cos\omega t = \sin 2\omega t$$

The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} -\frac{V_m I_m}{2}\sin(2\omega t)d(\omega t) = 0$$

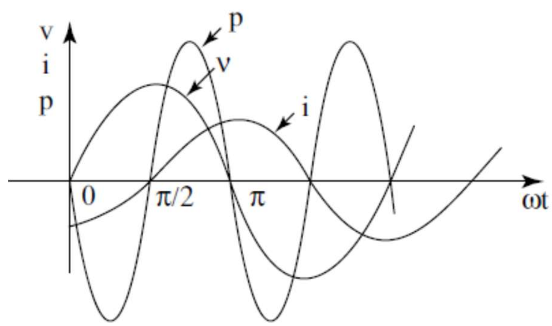


Fig 2.18 v, i, p waveform

It can be observed that when power curve is positive, energy gets stored in the magnetic field established due to increasing current while during negative power curve; this power is returned back to the supply.

The areas of positive loop and negative loop are exactly same and hence, average power consumption is zero.

A.C. THROUGH PURE CAPACITANCE

Consider a simple circuit consisting of a pure capacitor of C-farads, connected across a voltage given by the equation $v = V_m \sin\omega t$.

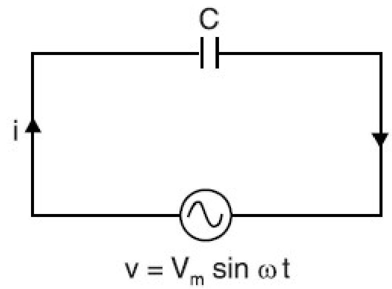


Fig 2.19 Pure Capacitive circuit

The current **i** charge the capacitor C. The instantaneous charge '**q**' on the plates of the capacitor is given by,

$$q = Cv$$
$$\therefore q = CV_m \sin\omega t$$

But, current is rate of flow of charge.

$$\therefore \quad i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\omega t)$$
$$\therefore \quad i = CV_m \frac{d}{dt}(\sin\omega t) = CV_m \omega \cos\omega t$$
$$\therefore \quad i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$
$$\therefore \quad i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where $\mathbf{I_m} = \frac{\mathbf{V_m}}{\mathbf{X_C}}$ and $\boldsymbol{X_C} = \frac{\mathbf{1}}{\boldsymbol{\omega C}} = \frac{\mathbf{1}}{\boldsymbol{2\pi f C}}$ **Ω**.

The term X_c is called **capacitive reactance** and measured in **ohms.**

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $\frac{\pi}{2}$ radians i.e, **90°**. This means **current leads voltage applied by 90°**.The positive sign indicates leading nature of the current.

Fig 2.20 shows waveform of voltage and current and the corresponding phasor diagram. The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

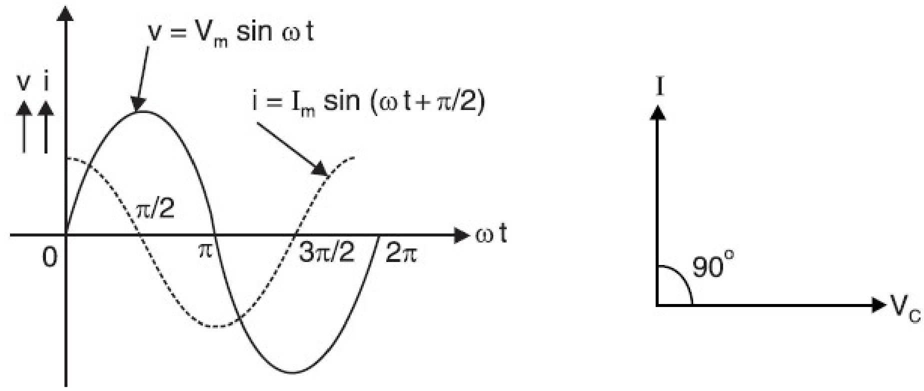


Fig 2.20 Voltage and Current Waveform and Corresponding phasor diagram

Power in pure capacitive circuit

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$\mathrm{p} = \mathbf{v} \times \mathbf{i} = V_{\mathrm{m}}\sin\omega t \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin(\omega t)\cos(\omega t) \qquad \because \sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t$$

$$\mathbf{p} = \frac{V_{\mathrm{m}} I_m}{2}\sin(2\omega t) \qquad \because 2\sin\omega t\cos\omega t = \sin 2\omega t$$

The average value of sine curve over a complete cycle is always zero.

$$P_{\mathrm{av}} = P = \int_0^{2\pi} \frac{\mathrm{V_m I_m}}{2}\sin(2\omega \mathrm{t})\mathrm{d}(\omega \mathrm{t}) = 0.$$

Fig 2.21 shows waveforms of current, voltage and power. It can be observed from the figure that when power curve is positive, in practice, an electrostatic energy gets stored in the capacitor during its charging while the negative power curve represents that the energy stored is returned back to the supply during its discharging. The areas of positive and negative loops are exactly the same and hence, average power consumption is zero.

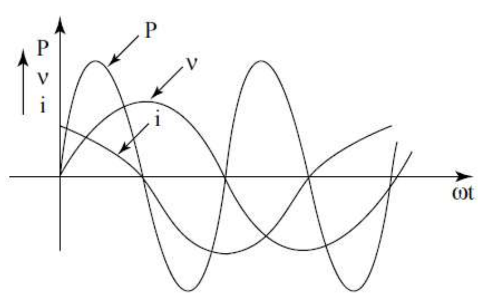


Fig 2.21 v, i, p waveform

A.C.THROUGH SERIES R-L CIRCUIT

Consider a circuit consisting of pure resistance **R** ohms connected in series with a pure inductance of **L** henries.

The series combination is connected across a.c. supply given by $v = V_m \sin\omega t$.

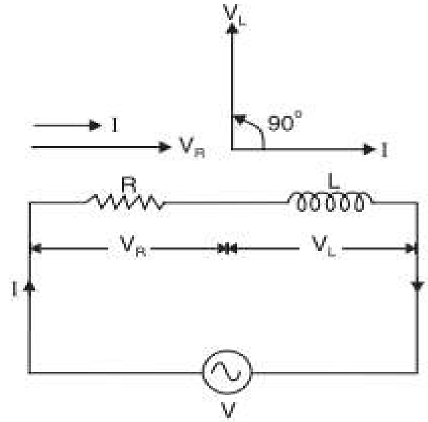


Fig 2.22 Series R L Circuit

Circuit draws a current **I** then there are two voltage drops,

a) Drop across pure resistance, $V_R = IR$

b) Drop across pure inductance, $V_L = I\,X_L$ where $X_L = 2\pi f\,L$

I = r.m.s. value of current drawn

V_R, V_L = r.m.s. values of voltage drops

The Kirchhoff's law can be applied to the a.c. circuit but only the point to remember is the addition of voltages should be vector addition.

$$\vec{V} = \overrightarrow{V_R} + \overrightarrow{V_L}$$
$$\vec{V} = \vec{I}R + \vec{I}X_L$$

The phasor diagram and the voltage triangle for the above case are shown in Fig 2.23 & 2.24 respectively.

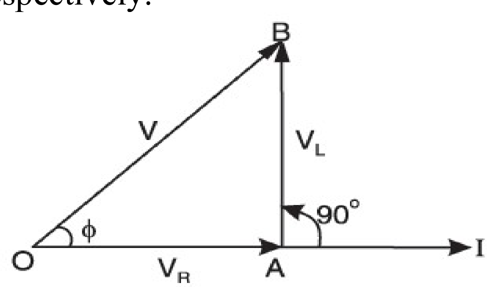


Fig 2.23 Phasor Diagram

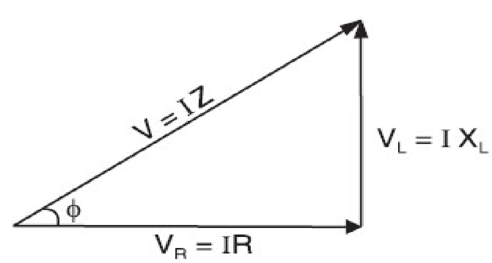


Fig 2.24 Voltage Triangle

From the voltage triangle

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called impedance of the circuit. It is represented by |Z| and is measured in Ohms.

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{|\boldsymbol{Z}|} \text{ where } |\boldsymbol{Z}| = \sqrt{\boldsymbol{R^2} + \boldsymbol{X_L^2}}$$

Also, from the phasor diagram, it is evident that, the current flowing through R-L series circuit lags the supply voltage by an angle of $\boldsymbol{\phi}$ degrees. Hence the instantaneous value of current can be expressed as,

$$\mathbf{i} = \mathbf{I_m}\sin(\omega\mathbf{t} - \phi)$$

IMPEDANCE OF SERIES R-L CIRCUIT

Impedance is defined as the opposition of circuit to the flow of alternating current. It is denoted by Z and its unit is ohms.

From the voltage triangle, we can write

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\text{Cos}\,\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{\text{R}}{|Z|}$$

$$\text{Sin}\phi = \frac{V_L}{V} = \frac{IX_L}{IZ} = \frac{X_L}{|Z|}$$

If all the sides of the voltage triangle are divided by current, we get a triangle called impedance triangle as shown in Fig 2.25.

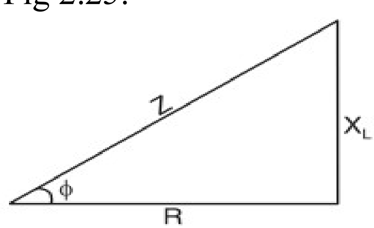


Fig 2.25 Impedance Triangle

From the impedance triangle, we can see that the X component of impedance is R and is given by **R=|Z|cosφ** and Y component of impedance is X_L and is given by $\mathbf{X_L=|Z|\sin\varphi}$. In rectangular form the impedance is denoted as

$$Z = R + jX_L$$

While in the polar form, it is denoted as

$$Z = |Z|\angle\phi$$

$$|Z| = \sqrt{R^2 + X_L^2}, \qquad \phi = \tan^{-1}\left[\frac{X_L}{R}\right]$$

POWER AND POWER TRIANGLE IN SERIES R-L CIRCUIT

The expression for the current in a series R-L circuit is

$$\mathbf{i} = \mathbf{I_m}\sin(\omega\mathbf{t} - \phi)$$

Instantaneous power

$$p = vi = V_m\sin(\omega t) \times I_m\sin(\omega t - \phi)$$

$$= \mathbf{V}_m I_m[\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m I_m\left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

Instantaneous Power consists of two terms

(a) Constant part $\frac{\boldsymbol{V_m I_m}}{\mathbf{2}}\boldsymbol{\cos\phi}$ whose average value over a cycle is the same.

(b) A pulsating component $\frac{\boldsymbol{V_m I_m}}{\mathbf{2}}\boldsymbol{\cos(2\omega t - \phi)}$ whose average value over one complete cycle is zero.

$$\text{Average power, } P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$$

$$\boldsymbol{P} = VI\boldsymbol{\cos\phi}$$

where V and I are the r.m.s. values of voltage and current.

The term $\boldsymbol{\cos\phi}$ is called power factor of the circuit and its value is given by;

$$\text{Power factor(p.f), } \boldsymbol{\cos\phi} = \frac{\boldsymbol{IR}}{\boldsymbol{IZ}} = \frac{\boldsymbol{R}}{|\boldsymbol{Z}|}$$

If we multiply voltage equation by current, we get the power equation

$$\overrightarrow{\mathrm{VI}} = \overrightarrow{\mathrm{V_R}I} + \overrightarrow{\mathrm{V_L}I}$$

$$\overrightarrow{VI} = \vec{V}\cos\phi\vec{I} + \vec{V}\sin\phi\vec{I}$$

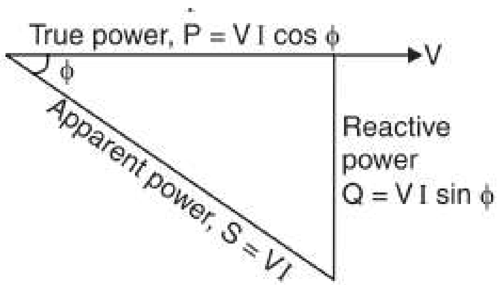


Fig 2.26 Power Triangle

Fom this equation, Power Triangle can be drawn as shown in the Fig 2.26. Three sides of this triangle are

a) VI
b) $VI\cos\phi$
c) $VI\sin\phi$

APPARENT POWER(S)

The total power that appears to be transferred between the source and load is called apparent power. It is equal to the product of applied voltage (V) and circuit current (I)

i.e, **Apparent power,** $\boldsymbol{S = V \times I = VI}$

It is measured in **volt-ampers (VA)**. Apparent power has two components: true power and reactive power.

TRUE POWER(P)

The power which is actually consumed in the circuit is called true power or active power. We know that power is consumed in resistance only since neither pure inductor (L) nor pure capacitor (C) consumes any active power. Now, current and voltage are in phase in a resistance. Therefore, current in phase with voltage produces true or active power. It is the useful component of apparent power.

The product of voltage (V) and component of total current in phase with voltage ($\boldsymbol{I\cos\phi}$) is equal to true power.

i.e, True Power = Voltage × Component of total current in phase with voltage

$$\boldsymbol{= V \times I\cos\phi}$$

$$\boldsymbol{P = VI\cos\phi}$$

It is measured in **watts (W)**.

REACTIVE POWER(Q)

The component of apparent power which is neither consumed nor does any useful work in the circuit is called reactive power.

The product of voltage (V) and component of total current 90° out of phase with voltage ($\boldsymbol{I\sin\phi}$) is equal to reactive power.

i.e, Reactive power, Q = Voltage × Component of total current 90° out of phase with voltage

$$\boldsymbol{= V \times I\sin\phi}$$

$$\boldsymbol{Q = VI\sin\phi}$$

It is measured in **volt-amperes reactive (VAR)**.

POWER FACTOR

It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.

It is the ratio of true power to apparent power.

$$\textbf{Power factor} = \frac{VI\cos\phi}{VI} = \frac{\textbf{True power}}{\textbf{Apparent power}}$$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It cannot be greater than1.

It is also defined as the ratio of resistance to the Impedance.

$$\textbf{Power factor} = \frac{R}{|Z|} = \frac{\textbf{Resistance}}{\textbf{Impedance}}$$

If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.

So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. ϕ=0.Therefore, power factor is cos(0°)=1. Such circuit is called unity power factor circuit.

$$\text{Power factor} = \cos\phi = \text{ cosine of angle between } V \text{ and } I$$

SIGNIFICANCE OF POWER FACTOR

The apparent power drawn by a circuit has two components viz. (i) true power and (ii) reactive power. True power component should be as large as possible because it does useful work in the circuit. This is possible only if the reactive power component is small. Smaller the phase angle ϕ (i.e. greater the p.f. cosϕ), the smaller is the reactive power component. Thus when ϕ = 0° (i.e. cosϕ = 1), the reactive power component is zero and the true power is equal to the apparent power. That means the whole of apparent power drawn by the circuit is being utilized by it.

Thus, power factor of a circuit is a measure of its effectiveness in utilizing the apparent power drawn by it. The greater the power factor of a circuit, the greater is its ability to utilize the apparent power. Thus 0·5 p.f. (i.e. 50% p.f.) of a circuit means that it will utilize only 50% of the apparent power whereas 0·8 p.f. would mean 80% utilization of apparent power. For this reason, the power factor of the circuit should be as near to 1 as possible.

A.C. THROUGH SERIES R-C CIRCUIT

Consider a circuit consisting of pure resistance **R**-ohms and connected in series with a pure capacitor of **C**-Farads as shown in the Fig. 2.27. The series combination is connected across a.c. supply given by $v=V_m \sin\omega t$.

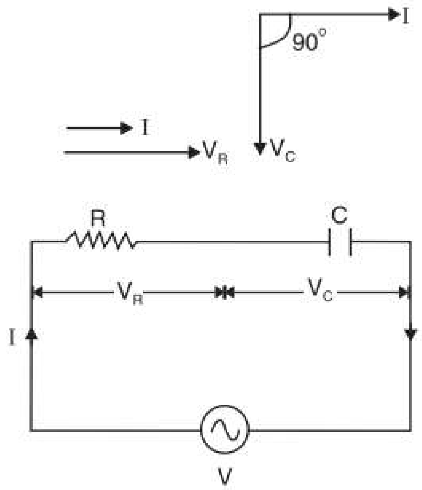


Fig 2.27 Series RC Circuit

Circuit draws a current **I**, then there are two voltage drops,

a. Drop across pure resistance, $\mathbf{V_R = I\,R}$

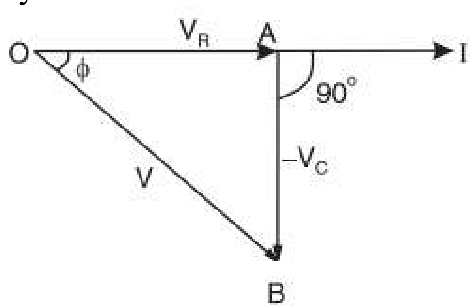
b. Drop across pure capacitance, $\mathbf{V_C} = \boldsymbol{I}\mathbf{X_C}$ where $\mathbf{X_C} = \frac{\mathbf{1}}{\mathbf{2\pi fC}}$

I= r.m.s. value of current drawn, V_R, V_c= r.m.s. values of voltage drops

The Kirchoff's law can be applied to the a.c. circuit but only the point to remember is the addition of voltages should be vector addition.

$$\vec{V} = \overrightarrow{V_R} + \overrightarrow{V_C}$$
$$\vec{V} = \vec{I}R + \vec{I}X_C$$

The phasor diagram and the voltage triangle for the above case are shown in Fig 2.28 & 2.29 respectively.



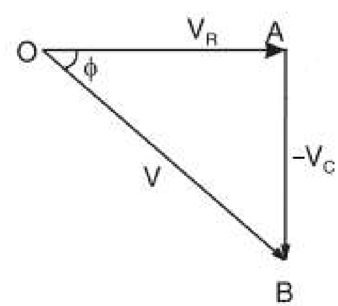


Fig 2.28 Phasor Diagram **Fig 2.29 Voltage Triangle**

From the voltage triangle,

$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{R^2 + X_C^2}$ offers opposition to current flow and is called impedance of the circuit.

$$\boldsymbol{I = \frac{V}{|Z|}} \text{ where } \boldsymbol{|Z| = \sqrt{R^2 + X_C^2}}$$

Also, from the phasor diagram, it is evident that, the current flowing through R-C series circuit leads the supply voltage by an angle of $\boldsymbol{\phi}$ degrees. Hence the instantaneous value of current can be expressed as,

$$\mathbf{i} = \mathbf{I_m}\sin(\omega\mathbf{t} + \phi)$$

IMPEDANCE OF SERIES R-C CIRCUIT

Similar to R-L series circuit, the impedance is nothing but the opposition to the flow of alternating current. It is measured in ohms given by

$$|Z| = \sqrt{R^2 + X_C^2}$$

From the voltage triangle, we can write

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

$$\text{Cos}\,\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{\text{R}}{Z}$$

$$\text{Sin}\phi = \frac{V_C}{V} = \frac{IX_C}{IZ} = \frac{X_C}{Z}$$

If all the sides of the voltage triangle are divided by current, we get a triangle called impedance triangle as shown in Fig. 2.30.

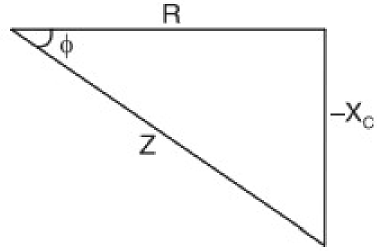


Fig 2.30 Impedance Triangle

From the impedance triangle, we can see that the X component of impedance is **R** and is given by **R=|Z|cosφ** and Y component of impedance is X_C and is given by X_C=**|Z|sinφ**.

In rectangular form the impedance is denoted as

$$\mathbf{Z = R - jX_C}$$

While in the polar form, it is denoted as

$$Z = |Z|\angle - \phi$$

$$|Z| = \sqrt{R^2 + X_C^2}, \qquad \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$$

where V and I are the r.m.s. values of voltage and current.

The term $\boldsymbol{\cos\phi}$ is called power factor of the circuit and its value is given by;

Power factor, $\boldsymbol{\cos\phi = \frac{IR}{IZ} = \frac{R}{|Z|}}$

If we multiply voltage equation by current we get the power equation

$$\overrightarrow{VI} = \overrightarrow{V_R I} + \overrightarrow{V_C I}$$

$$\overrightarrow{VI} = \vec{V}\cos\phi\vec{I} + \vec{V}\sin\phi\vec{I}$$

POWER IN SERIES R-C CIRCUIT

The expression for the current in a series R-C circuit is

$$\mathbf{i} = \mathbf{I_m}\sin(\omega\mathbf{t} + \phi)$$

Instantaneous power

$$p = vi = V_m\sin(\omega t) \times I_m\sin(\omega t + \phi)$$

$$= \mathbf{V}_m I_m[\sin(\omega t) \cdot \sin(\omega t + \phi)]$$

$$= V_m I_m\left[\frac{\cos(\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

Instantaneous Power consists of two terms

(a) Constant part $\boldsymbol{\frac{V_m I_m}{2}\cos\phi}$ whose average value over a cycle is the same.

(b) A pulsating component $\boldsymbol{\frac{V_m I_m}{2}\cos(2\omega t + \phi)}$ whose average value over one complete cycle is zero.

Average power, $P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$

$$\boldsymbol{P = VI\cos\phi}$$

A.C. THROUGH SERIES R-L-C CIRCUIT

Consider a circuit consisting of resistance **R** Ohms, pure Inductance **L** Henries and capacitance **C** Farads connected in series with each other across a.c. supply. The circuit is as shown below.

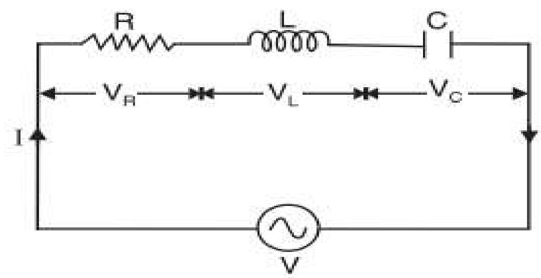


Fig 2.31 Series RLC circuit

The a.c. supply is given by, $\boldsymbol{v = V_m \, sin\omega t}$

The circuit draws a current **I**. Due to current **I**, there are different voltage drops across R, L and C which are given by,

a. Voltage across R, V_R = I R ... V_R is in phase with I.
b. Voltage across L, V_L = I X_L ... where V_L leads I by 90°.
c. Voltage across C, V_C = I X_C ... where V_C lags I by 90°.

According to Kirchoff's law we can write

$$\vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C$$

Following are the steps to draw the phasor diagram: Take current as reference.

a) V_R is in phase with I.

b) V_L leads current I by 90°.

c) V_C lags current I by 90°.

d) Obtain the resultant of V_L and V_C. Both V_L and V_C are 180° out of phase.

e) Add that with V_R by law of parallelogram to get the supply voltage.

The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C. Let us consider the different cases.

Case(i): $\boldsymbol{X_L > X_c}$

When $X_L > X_C$, obviously, I X_L i.e. V_L is greater than I X_C i.e. V_C. So, resultant of V_L and V_C will be directed towards V_L i.e. leading current I. Current I will lag the resultant of V_L and V_C i.e. $(V_L - V_C)$ by 90°.

The circuit is said to be inductive in nature. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. **Current I lags the voltage V by $\boldsymbol{\phi}$**. The phasor diagram and the voltage triangle for the above case are shown in Fig 2.32 & 2.33 respectively.

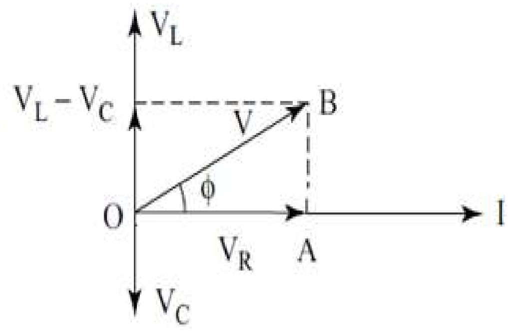


Fig 2.32 Phasor Diagram

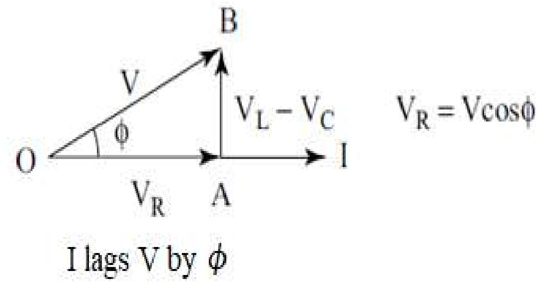


Fig 2.33 Voltage Triangle

From the Voltage Triangle

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$I = I|Z|$$

Where $|\mathbf{Z}| = \sqrt{\mathbf{R}^2 + (\mathbf{X_L} - \mathbf{X_C})^2}$

So, if $\boldsymbol{v = V_m sin\omega t}$, then $\boldsymbol{i = I_m sin(\omega t - \phi)}$ as current lags voltage by angle $\boldsymbol{\phi}$.

Case(ii): $\mathbf{X_L < X_C}$

When $\mathbf{X_L < X_C}$, obviously, IX_L i.e V_L is less than IX_C i.e V_C. So the resultant of V_L and V_C will be directed towards V_C. Current I will lead $(V_C - V_L)$ by 90°.

The current is said to be capacitive in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. **Current I leads the voltage V by $\boldsymbol{\phi}$**. The phasor diagram and the voltage triangle for the above case are shown in Fig 2.34 & 2.35 respectively.

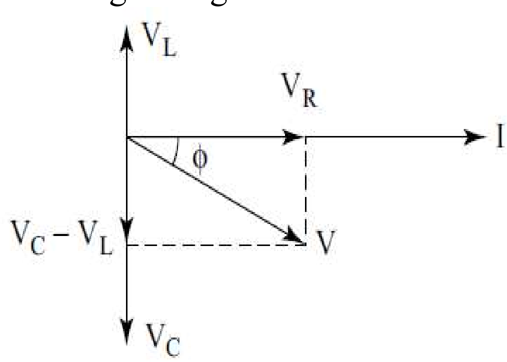


Fig 2.34 Phasor Diagram

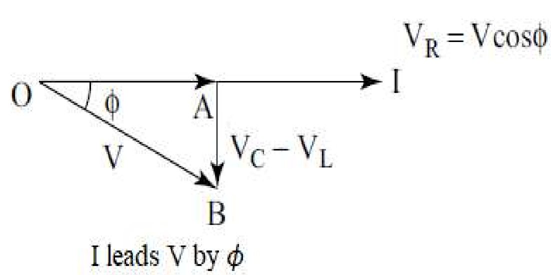


Fig 2.35 Voltage Triangle

Case (iii): X_L=X_C

When $X_L = X_C$, obviously $V_L = V_C$. So V_L and V_C will cancel each other and their resultant is zero.

So, $\mathbf{V_R} = \mathbf{V}$ in such case and overall circuit is purely resistive in nature. The phasor diagram for the above case is shown in Fig 2.36

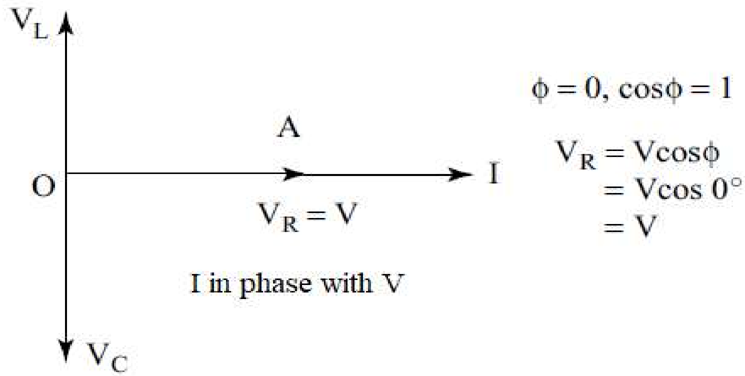


Fig 2.36 Phasor Diagram

From the phasor diagram V=$\mathbf{V_R}$ = IR= IZ where Z=R The circuit is purely resistive with unity power factor.

Impedance of series RLC circuit

In general, for RLC series circuit impedance is given by,

$$Z = R \pm jX$$

Where $X = X_L - X_C$ = total reactance of circuit.
If $X_L > X_C$, X is positive and circuit is inductive.
If $X_L < X_C$, X is negative and circuit is capacitive.
If $X_L = X_C$, X is zero and circuit is purely resistive.

$$\tan\phi = \{\frac{X_L - X_C}{R}\} \quad \cos\phi = \frac{R}{|Z|} \text{ and } \quad |Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance Triangle of series RLC circuit

The impedance is expressed as,

$$Z = R \pm jX \text{ where } X = X_L - X_C$$

For $X_L > X_C$, $(X_L - X_C)$ is positive, so ϕ is positive and the impedance triangle is as shown in the Fig 2.37(a). Similarly for $X_L < X_C$, $(X_L - X_C)$ is negative, so ϕ is negative and the impedance triangle is as shown in Fig 2.37(b).

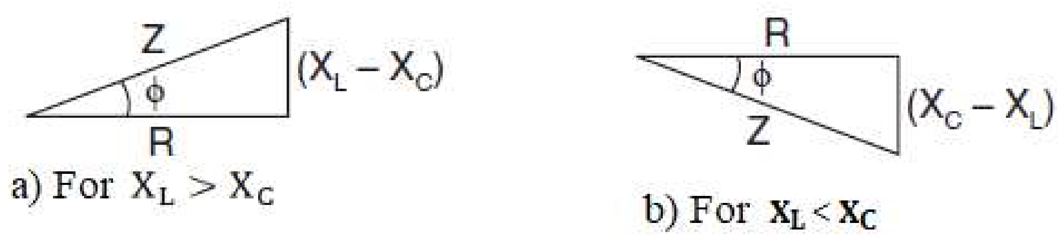


Fig 2.37 Impedance Triangle

In both the cases, $\mathbf{R = Zcos\phi}$ and $\mathbf{X = Zsin\phi}$.

Power in series RLC circuit

The average power consumed by the circuit

P = Average power consumed by R+ Average power consumed by L + Average power consumed by C.

But, pure L and C never consume any power.

$$\therefore \text{P} = \text{ Power taken by } R = I^2R = I(IR) = IV_R = VICos\phi$$

Since, $V_{\mathbf{R}} = \mathbf{V}\cos\phi$ in both the cases.

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