# MODULE 3
# Three Phase AC Circuits and Transformers

**Syllabus:**
**Three-phase AC Circuits**: Generation of three-phase voltage; Representation of balanced star and delta system (for both source and load); the relation between phase and line values of voltage and current.
**Single-phase Transformer**: Working principle, Types of Transformer, Transformation ratio, EMF Equation.

## Three Phase AC Circuits: Introduction

- A single phase a.c. voltage can be generated by rotating a coil of one turn made up of two conductors, in a magnetic field. Such an a.c. producing machine is called single turn alternator.
- But voltage produced by such a single turn is very less and not enough to supply practical loads.
- Hence number of turns are connected in series to form one winding in a practical alternator, such a winding is called armature winding.
- The sum of the voltages induced in all the turns is now available as a single phase a.c. voltage, which is sufficient to drive the practical loads.
- But in practice there are certain loads which require polyphase supply.
- Phase means branch, circuit or winding while poly means many. So such applications need a supply having many a.c. voltages present in it simultaneously. Such a system is called polyphase system.
- To develop polyphase system, the armature winding in an alternator is divided into number of phases required.
- In each section, a separate a.c. voltage gets induced. So there are many independent ac.voltages present equal to number of phases of armature winding.
- The various phases of armature winding are arranged in such a manner that the magnitudes and frequencies of all these voltages is same but they have definite phase difference with respect to each other.
- The phase difference depends on number of phases in which armature is divided.
- For example, if armature is divided into three coils then three separate a.c. voltages will be available having same magnitude and frequency but they will have a phase difference of 360°/3 = 120° with respect to each other.
- All three voltages with a phase difference of 120° are available to supply a three phase load.
- Such a supply system is called three phase system. Similarly by dividing armature into various numbers of phases, a 2 phase, 6 phase supply system also can be obtained.
- A phase difference between such voltages Is 360°/n where n is number of phases.

**Advantages of Three Phase System**

In the three phase system, the alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase difference of 120° between each other. Such a three phase system has following advantages over single phase system.

1. The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2. For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage ratingso transmission becomes very much economical.
3. It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.
4. In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.
5. Three phase system give steady output.
6. Single phase supply can be obtained from three phase but three phase cannot be obtainedfrom single phase.
7. Power factor of single phase motors is poor than three phase motors of same rating.
8. For converting machines like rectifiers, the D.C. output voltage becomes smoother if number of phases is increased.

But it is found that optimum number of phases required to get all above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard system throughout the world.

**Generation of Three Phase Voltage System**

An alternator consisting of one group of coils on armature produces one alternating voltage. But if armature coils are divided into three groups such that they are displaced by the angle 120° from each other, three separate alternating voltages get developed

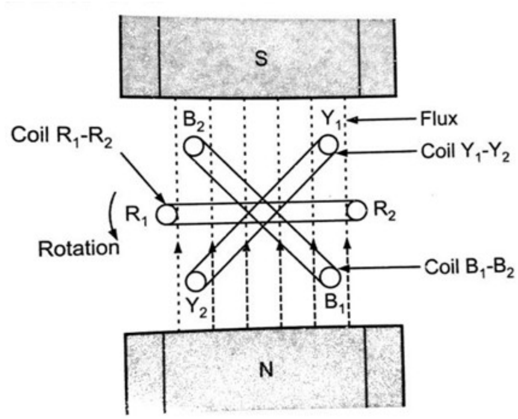


Fig 3.1: Generation of 3-phase voltage

- Consider armature of alternator divided into three groups as shown in the Fig. 3.1.
- The coils are named as R1- R2, Y1 -Y2 and B1- B2 and mounted on same shaft.
- The ends of each coil are brought out through the slip ring and brush arrangement to collect the Induced e.m.f.
- Let eR, eY and eB be the three independent voltages in coil R1-R2, Y1-Y2 and B1-B2 respectively.
- All are alternating voltages having same magnitude and frequency as they are rotated at uniform speed.
- All of them will be displaced by one another by 120°.
- Suppose $e_R$ is assumed to be reference and is zero for the instant shown in Fig 3.2.

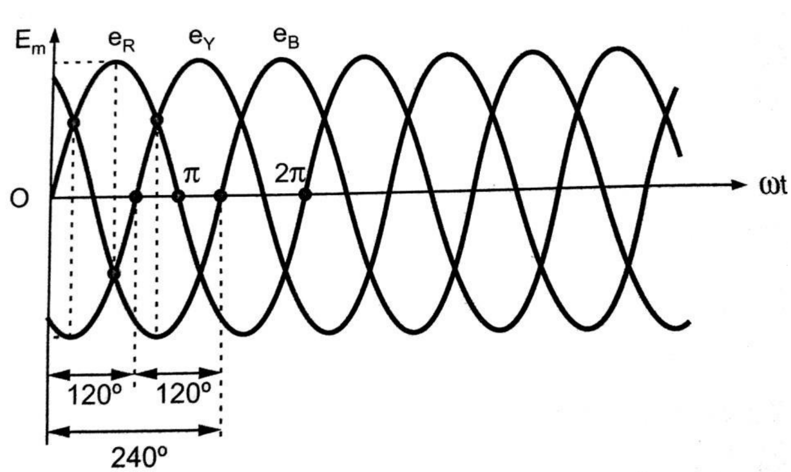


Fig 3.2 Phase voltages

- At the same instant $e_Y$ will be displaced by 120° from $e_R$ and will follow $e_R$ while $e_B$ will be displaced by 120° from $e_Y$ and will follow $e_Y$.
- All coils together represent three phase supply system.
- The equation of the induced e.m.f are

$$e_R = E_m \sin(\omega t)$$
$$e_Y = E_m \sin(\omega t - 120°)$$
$$e_B = E_m \sin(\omega t - 240°)$$

- The phasor diagram is shown in Fig 3.3

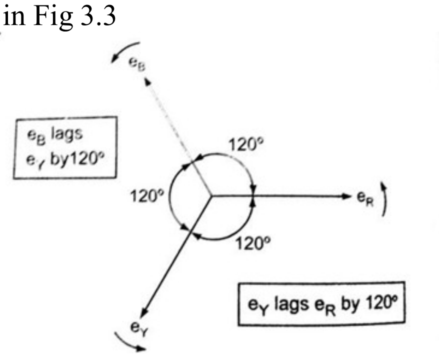


Fig. 3.3 Phasor Diagram

- If the three voltages are added vectorially, it can be observed that the sum of these three voltages at any instant is zero.

Mathematically this can be shown as :

$$e_R + e_Y + e_B$$

$$= E_m \sin \omega t + E_m \sin(\omega t - 120°) + E_m \sin(\omega t + 120°)$$

$$= E_m [\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 120° + \cos \omega t \sin 120°]$$

$$= E_m [\sin \omega t + 2 \sin \omega t \cos 120°] = E_m \left[\sin \omega t + 2 \sin \omega t \left(\frac{-1}{2}\right)\right] = 0$$

$$\therefore \quad \boxed{\bar{e}_R + \bar{e}_Y + \bar{e}_B = 0}$$

**Phase Sequence**

The sequence in which the voltages in three phases reach their maximum positive values iscalled phase-sequence. Generally the phase sequence is R-Y-B.

The significance of the phase sequence of the three phase supply is:

1. When the 3 phase supply of a particular sequence is given to a static three phase load, certain current flows through the line and phase of the load. If the phase sequence is changed, then both magnitude and phase of the currents flowing in the lines and the phase of the load will change.
2. If the load is a three phase induction motor, when the sequence of the supply is changed,not only the magnitude and phase of the line current and phase current change, but the direction of rotation of motor also changes.

**Three Phase Supply Connections**

In single phase system, two wires are sufficient for transmitting voltage to the load i.e. phase and neutral. But in case of three phase system, two ends of each phase i.e. R1-R2, Y1-Y2 andB1-B2 are available to supply voltage to the load. If all six terminals are used independently to supply voltage to load as shown in the Fig, 3.4 then total six wires will be required and it will be very much costly.

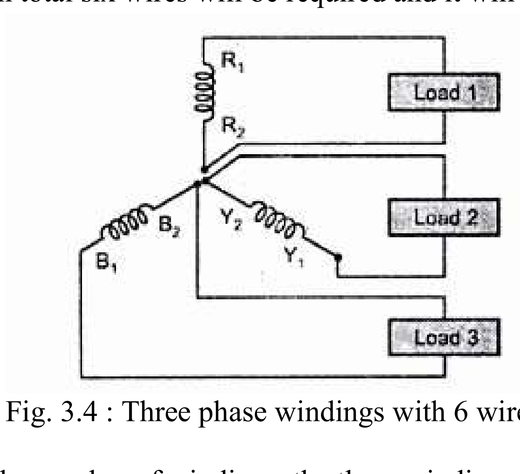


Fig. 3.4 : Three phase windings with 6 wires

To reduce the cost by reducing the number of windings, the three windings are interconnected in a particular fashion . This gives different three phase connections.

**Star Connection**

The star connection is formed by connecting starting or terminating ends of all the three windings together. The ends R1 -Y1 -B1 are connected or ends R2 -Y2 -B2 are connected together. This common point is called Neutral Point. The remaining three ends are brought outfor connection purpose. These ends are generally referred as R-Y-B, to which load is to be connected. (Fig. 3.5)

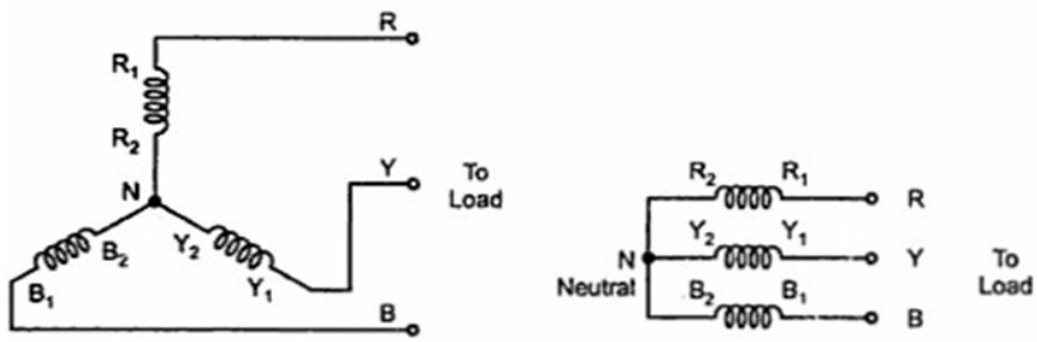


Fig 3.5 Star connection

**Delta Connection**

The delta is formed by connecting one end of winding to starting end of other and connections are continued to form a dosed loop. The supply terminals are taken out from the three junction points. The delta connection is shown in the Fig. 3.6

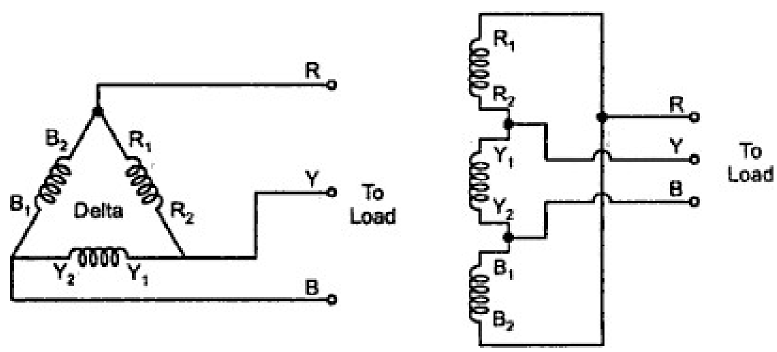


Fig. 3.6 Delta Connection

**Concept of Line Voltages and Line Currents**

The potential difference between any two lines of supply is called line voltage and current passing through any line is called line current. Consider a star connected system as shown in the Fig. 3.7

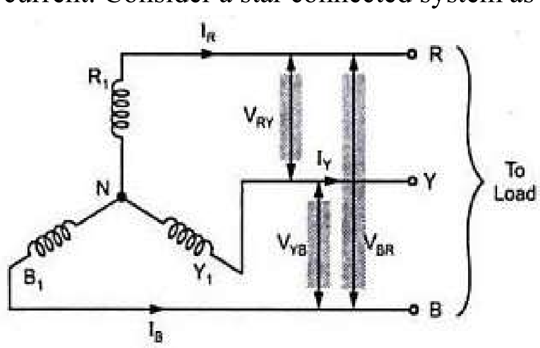


Fig. 3.7 Star connected system with line voltages and line currents

Line voltages are denoted by $V_L$. These are $V_{RY}$, $V_{YB}$ and $V_{BR}$.
Line currents are denoted by $I_L$.These are $I_R$, $I_Y$ and $I_B$
Similarly for delta connected system we can show the Line voltages and line currents as in the Fig. 3.8

Line voltages $V_L$ are $V_{RY}$, $V_{YB}$ and $V_{BR}$.

While Line currents $I_L$ are $I_R$, $I_Y$ and $I_B$.

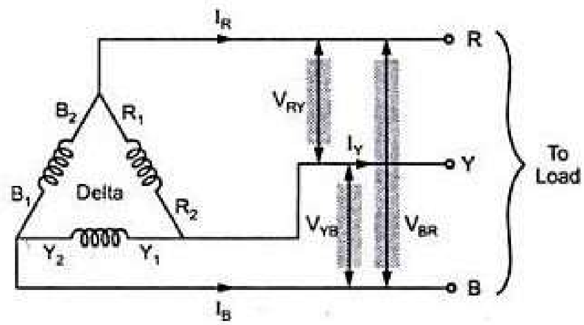


Fig. 3.8 Delta connected system with Line voltages and Line currents

## 3-phase Star and Delta connected loads

Generally Red, Yellow and Blue coloured wires are used to differentiate three phases and hence the names given to three phases are R, Y and B**.**

The load can be connected in two ways, i) Star connection, ii) Delta connection

The three phase load is nothing but three different impedances connected together in star or delta fashion

1. **Star connected load**: There are three different impedances and are connected such thatone end of each is connected together and other three are connected to supply terminalsR-Y-B. This is shown in the Fig. 3.9

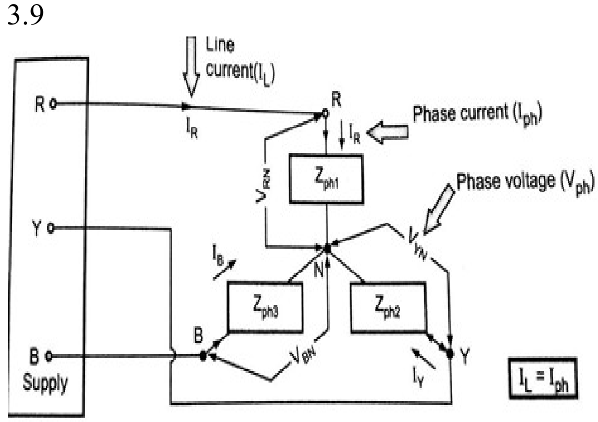


Fig. 3.9 Star connected load

- In the diagram shown $V_{RN}$, $V_{YN}$ and $V_{BN}$ are the phase voltages while $I_R$, $I_Y$ and $I_B$ are phase currents.
- The phase voltages are denoted as $V_{ph}$ while the phase currents are denoted as $I_{ph}$.

- Generally suffix N is not indicated for phase voltages in star connected load. So, $V_{ph}=V_R=V_Y=V_B$.
- It can be seen from the diagram that $I_{ph}=I_R=I_Y=I_B$.
- But same are the currents flowing in the three lines also. Thus we can conclude that for star connection

$$I_L = I_{ph}$$

$$V_L = \sqrt{3}\,Vph$$

2. **Delta Connection:** If the three impedances are connected such that the starting end of one is connected to the terminating end of other, to form a closed loop it is called deltaconnection of the load. The junction points are connected to supply terminals R-Y-B. This is shown in Fig. 3.10

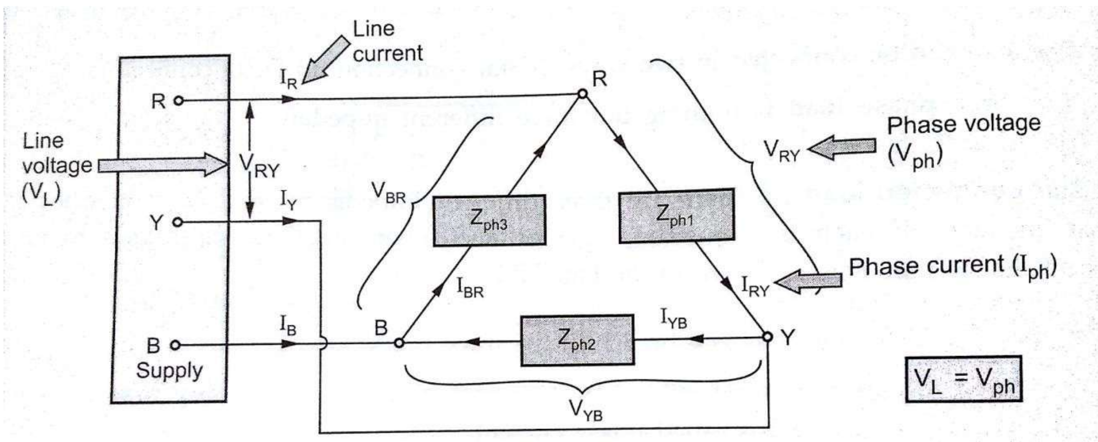


Fig. 3.10 Delta connected Load

- The currents $I_{RY}$, $I_{YB}$ and $I_{BR}$ flowing through the various branches of the load are phase currents. The line currents are $I_R$, $I_Y$ and $I_B$ flowing through supply lines. Thusin delta connection of the load, line and phase currents are different.
- The voltages across $Z_{ph1} = V_{RY}$, across $Z_{ph2} = V_{YB}$ and across $Z_{ph3} = V_{BR}$ and all arephase voltages.

$$V_{ph} = V_{RY} = V_{YB} = V_{BR}$$

- But as per definition of line voltages, same are the voltage across the supply line also. Thus, it can be concluded that in delta connection line voltage is equal to phase voltage.

$$V_L = V_{ph}$$

And $$I_L = \sqrt{3}\,Iph$$

**Power :** Power consumed in each phase is single phase power given by,

$$P_{ph} = V_{ph} I_{ph} \cos\phi$$

Total power $P = 3P_{ph} = 3V_{ph}I_{ph}\cos\phi = 3V_L \frac{I_L}{\sqrt{3}}\cos\phi$

$\therefore$ $P = \sqrt{3}V_L I_L \cos\phi$

### . Power Triangle for Three Phase Load

Total apparent power $S = 3\times$ Apparent power per phase

$\therefore$ $S = 3\,V_{ph}I_{ph} = 3\,\frac{V_L}{\sqrt{3}}I_L = 3\,V_L\frac{I_L}{\sqrt{3}}$

$\therefore$ $S = \sqrt{3}V_L I_L$ volt-amperes (VA) or kVA

Total active power $P = \sqrt{3}V_L I_L \cos\phi$ watts (W) or kW

Total reactive power $Q = \sqrt{3}V_L I_L \sin\phi$ reactive volt amperes (VAR) or kVAR

Hence power triangle is as shown in the Fig.

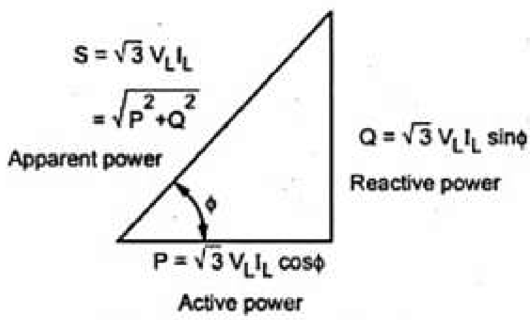


Fig 3.11 Power Triangle

# Transformers

## Introduction

Alternating voltages can be raised or lowered as per requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called transformer.

***The transformer is a static piece of apparatus by means of which an electrical power is transformed from one alternating current circuit to another with the desired change in voltage and current, without any change in the frequency.***

***It transfers AC-AC***

## Construction of single phase transformer

- There are two basic parts of transformer: i) Magnetic Core and ii) Winding or coils
- The core of the transformer is either square or rectangular in size.
- The core is further divided in to two parts. The vertical portion on which the coils are wound is called the ***limb*** while the top and bottom horizontal portion is called ***yoke*** of the core.
- The two windings (primary and secondary) of the transformer are insulated from each other and from the laminated steel core.
- The assembled core and windings are enclosed in a suitable container.
- Appropriate bushings, either of the porcelain or the capacitor type, are used for insulating and bringing out the terminals of the windings from the enclosure.
- The core is invariably constructed of transformer sheet laminations fitted together in such a way as to ensure a continuous magnetic path, with a minimum of air gap.
- The steel used for these laminations has high silicon content, which is sometimes heat treated to ensure high permeability and low hysteresis loss at the usual operating flux density.
- Lamination of the core minimises eddy current loss.
- These laminations are insulated from each other by a thin coating of suitable varnish.
- The diagram below (Fig. 3.11) shows the construction of a transformer

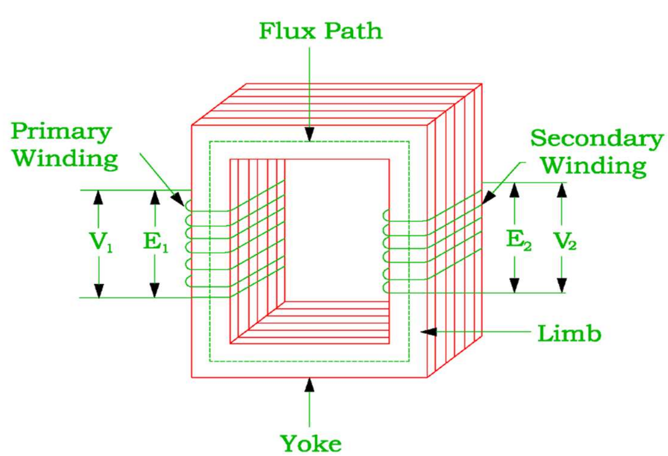


Fig 3.11 Construction of a transformer

## Principle of Working

A transformer works on the principle of ***mutual induction*** between the two coils. The general arrangement of the transformer is shown in Fig 3.12.

The coil into which electrical energy is fed is called primary winding (P), while the other coil from which electrical energy is drawn is called the secondary winding (S). The primary winding has $N_1$ number of turns while the secondary winding has $N_2$ number of turns.

When the primary winding is connected to an alternating voltage $V_1$, an alternating current flows through the primary winding P and this current produces an alternating flux φ in the steel core, the mean path of this flux being indicated by the dotted lines. If the entire flux produced by P passes through S, the EMF induced in each turn is the same for P and S.

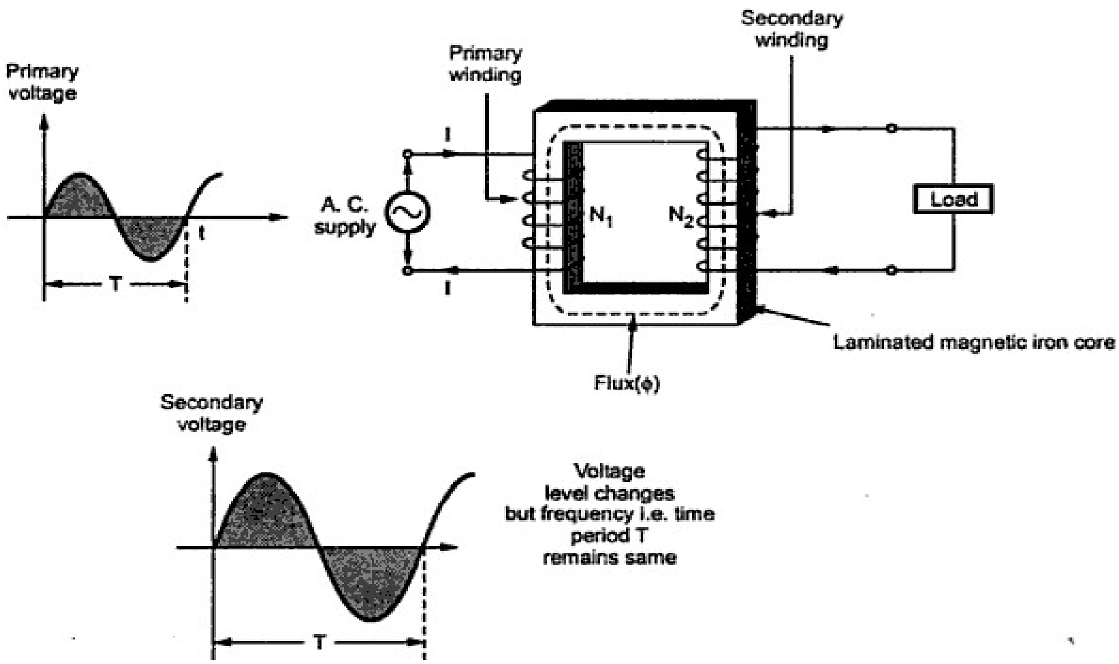


Fig 3.12 Working Principle of Transformer

The above mentioned alternating flux φ produces self induced EMF $E_1$ in the primary winding P, while due to mutual induction i.e., due to flux produced by primary linking the secondary, it produces mutually induced EMF $E_2$ in the secondary winding S.

Then EMFs are $E_1 = -N_1 \frac{d\varphi}{dt}$ and $E_2 = -N_2 \frac{d\varphi}{dt}$

Therefore $\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$

K is known as the ***voltage transformation ratio.***

The frequency of the two EMFs is the same. The voltage transformation ratio may be alternatively obtained as follows.

The EMF per turn is the same for P and S.

Hence,

$$\frac{\text{total EMF induced in S}}{\text{total EMF induced in P}} = \frac{N_2 * \text{EMF per turn}}{N_1 * \text{EMF per turn}} = \frac{N_2}{N_1} = K$$

When the secondary is on open circuit, its terminal voltage is the same as the induced EMF. The primary current is then small, so that the applied voltage $V_1$ is practically equal and opposite to the EMF induced in P. Hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = K \text{-------------------}(1)$$

As the full load efficiency of a transformer is almost 100%

$V_1 I_1 *$ Primary power factor = $V_2 I_2 *$ secondary power factor

As both the primary and secondary power factor are almost equal on full load

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = K \text{ ----------------------}(2)$$

From eqns (1) and (2)

$$\mathbf{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$$

## Types of transformers

Transformer is classified based on different parameters:

i)Based on construction

1. Core type
2. Shell type

ii) Based on Voltage level

1. Step-up transformer
2. Step-down transformer

**i) Based on Construction**

**1.Core type transformer**

- It has a single magnetic circuit.
- The core is rectangular having two limbs.
- The winding encircles the core.
- The coils used are of cylindrical type.
- The coils are wound in helical layer with different layers insulated from each other by paper or mica.
- Both the coils are placed on both the limbs.
- The low voltage coil is placed inside, near the core while the high voltage coil surrounds the low voltage coil.
- Core is made up of large number of thin laminations.
- As the windings are uniformly distributed over the two limbs the natural cooling is more effective.

- The coils can be easily removed by removing the lamination of the top yoke, for maintenance.
- Fig 3.13(a) shows the schematic representation of the core type transformer while 3.13(b) shows the view of actual construction of the core type transformer.

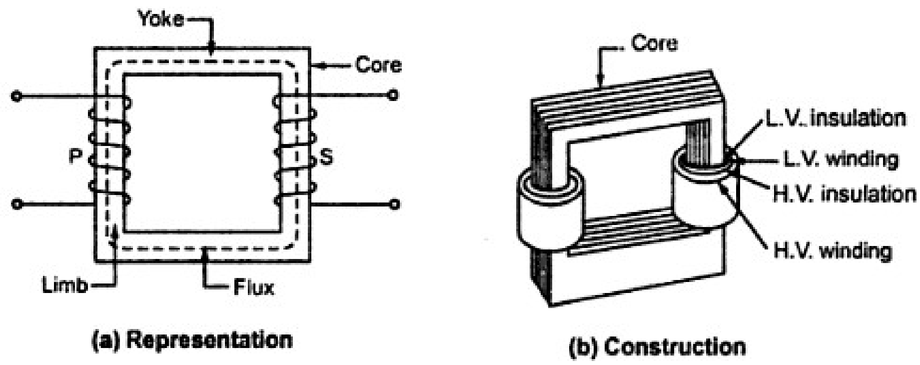


Fig 3.13 Core type transformer

## 2. Shell type transformer

- It has a double magnetic circuit.
- The core has three limbs.
- Both the windings are placed on the central limb.
- The core encircles most part of the windings.
- The coils used are generally multilayer disc type or sandwich coils.
- Each high voltage coil is in between low voltage coils and low voltage coils are nearest to top and bottom of the yokes.
- The core is laminated.
- While arranging the lamination of the core, the care is taken that all the joints at alternate layers are staggered.
- This is done to avoid narrow air gap at the joints, right through the cross section of the core. Such joints are called overlapped or imbricated joints.
- Generally for very high voltage transformers, the shell type construction is preferred.
- As the winding is surrounded by the core, the natural cooling does not exist.
- Fig 3.14(a) shows the schematic representation of the shell type transformer while 3.14(b) shows the view of actual construction of the shell type transformer

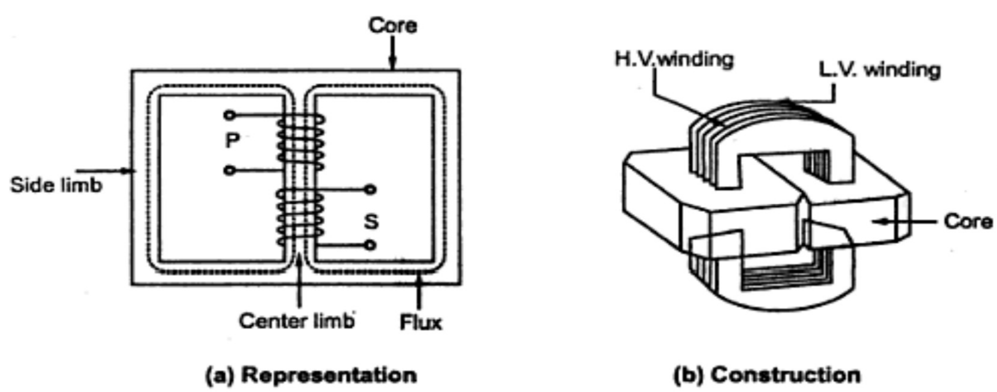


Fig 3.14 Shell type transformer

**ii) Based on Voltage level**

### 1. Step-up transformer

A transformer in which the output (secondary) voltage is greater than its input (primary) voltage is called a step-up transformer.

The step-up transformer decreases the output current for keeping the input and output power of the system equal.

Considere a step-up transformer shown in the figure 3.15 below. The $E_1$ and $E_2$ are the voltages, and $T_1$ and $T_2$ are the number of turns on the primary and secondary winding of the transformer.

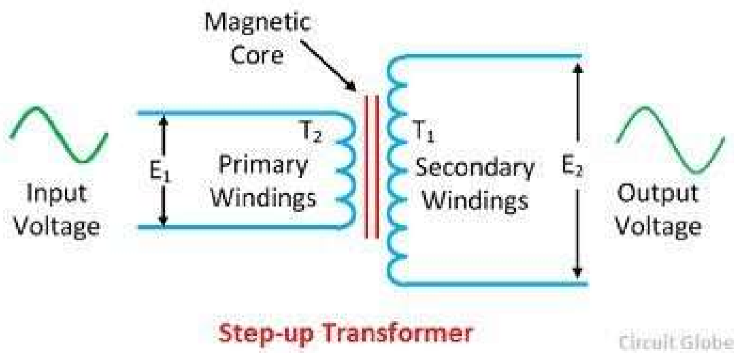


**Fig. 3.15 Step-up Transformer**

The number of turns on the secondary of the transformer is greater than that of the primary, i.e., $T_2 > T_1$.Thus the voltage turn ratio of the step-up transformer is 1:2. The primary winding of the step-up transformer is made up of thick insulated copper wire because the low magnitude current flows through it.

**Applications:** The step-up transformer is also used for starting the electrical motor, in the microwave oven, X-rays machines, etc.

### 2. Step-down transformer

A transformer in which the output (secondary) voltage is less than its input (primary) voltage is called a step-down transformer. The number of turns on the primary of the transformer is greater than the turn on the secondary of the transformer, i.e., $T_2 < T_1$. The step-down transformer is shown in the figure 3.16 below.

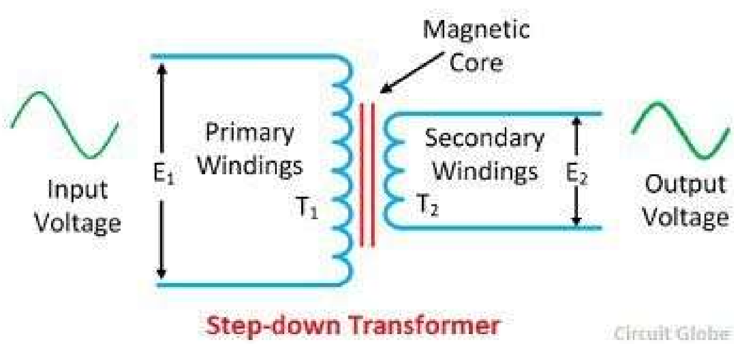


**Fig. 3.16 Step-down Transformer**

The voltage turn ratio of the step-down transformer is 2:1. The voltage turn ratio determines the magnitude of voltage transforms from primary to secondary windings of the transformer.

Step-down transformer is made up of two or more coil wound on the iron core of the transformer. It works on the principle of magnetic induction between the coils. The voltage applied to the primary of the coil magnetise the iron core which induces the secondary windings of the transformer. Thus the voltage transforms from primary to the secondary winding of the transformer.

**Applications** – It is used for electrical isolation, in a power distribution network, for controlling the home appliances, in a doorbell, etc.

## Toroidal transformer

A **toroidal transformer** is a type of electrical transformer constructed with a torus or donut-shaped core. Its primary and secondary windings are wound across the entire surface of the torus core, separated by an insulating material. This configuration minimizes the magnetic flux leakage. Therefore, a toroidal core is regarded as the ideal transformer core design. (Fig. 3.17)

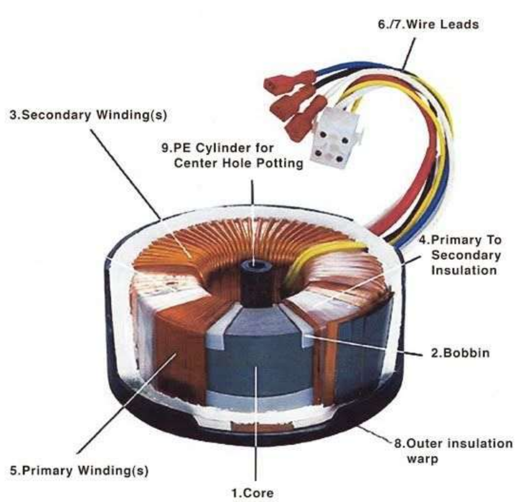


**Fig. 3.17 Toroidal Transformer**

Some of the advantages are high efficiency, quiet operation, minimal heat generation, and compact size. They are mostly seen in power supply systems, audio systems, control equipment, power inverters, and other electronic devices.

**Applications:**
- Isolation equipment for the medical industry
- Security systems
- Renewable energy (inverter systems)
- LED lighting
- Industrial control equipment
- Power distribution equipment
- Audio/visual equipment
- Automotive electronics
- Telecommunication systems

## EMF equation of a transformer

Consider a sinusoidally varying Voltage $V_1$ applied to the primary of the transformer shown in Fig 3.18.

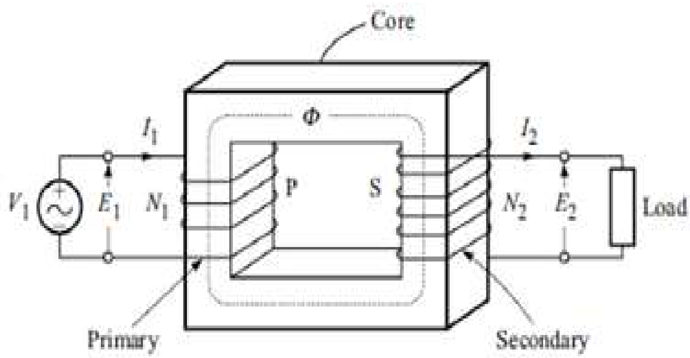


Fig 3.18 Transformer with voltages and currents

Due to this voltage, a sinusoidally varying magnetic flux is set up in the core which can be represented as

$$\Phi = \Phi_{\mathrm{m}}\sin\ \omega t = \Phi_{\mathrm{m}}\sin\ 2\pi f t$$

Where $\boldsymbol{\Phi_m}$ is the peak value of the flux and $\boldsymbol{f}$ is the frequency of sinusoidal variation of flux. As per the law of electromagnetic induction, the induced EMF in a winding of **N** turns is given as

$$e = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(\Phi_{\mathrm{m}}\sin\omega t)$$

$$= -N\omega\Phi_{\mathrm{m}}\cos\omega t$$

$$= \omega N\Phi_{\mathrm{m}}\sin\ (\omega t - \pi/2)$$

Thus, the peak of value of induced EMF is $\boldsymbol{E_{\mathrm{m}} = \omega N\Phi_{\mathrm{m}}}$. Therefore the RMS value of induced EMF is

$$E = \frac{E_{\mathrm{m}}}{\sqrt{2}} = \frac{\omega N\Phi_{\mathrm{m}}}{\sqrt{2}}$$

$$= \frac{2\pi f N\Phi_{\mathrm{m}}}{\sqrt{2}} = 4.44\mathrm{fN}\Phi_{\mathrm{m}}$$

$$\boldsymbol{E = 4.44\mathrm{fN}\Phi_{\mathrm{m}}}$$

This equation is known as the equation of EMF of transformer, which can be used to find the EMF induced in any winding (primary or secondary winding) linking with flux $\boldsymbol{\Phi}$**.**

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