MODULE 4

Electrical Machines: Generators and Motors

Syllabus:
Generator: Principle of operation, Constructional details, Types of generators and Applications
Motor: Principle of operation, Types of motors, Characteristics and Applications

4.1 Introduction

- An electrical machine, deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form. This process is called **electromechanical energy conversion**.
- An electrical machine which converts mechanical energy into an electrical energy is called an **electric generator**.
- An electrical machine which converts electrical energy into a mechanical energy is called an **electric motor.**

4.2 Types of Electrical Machines

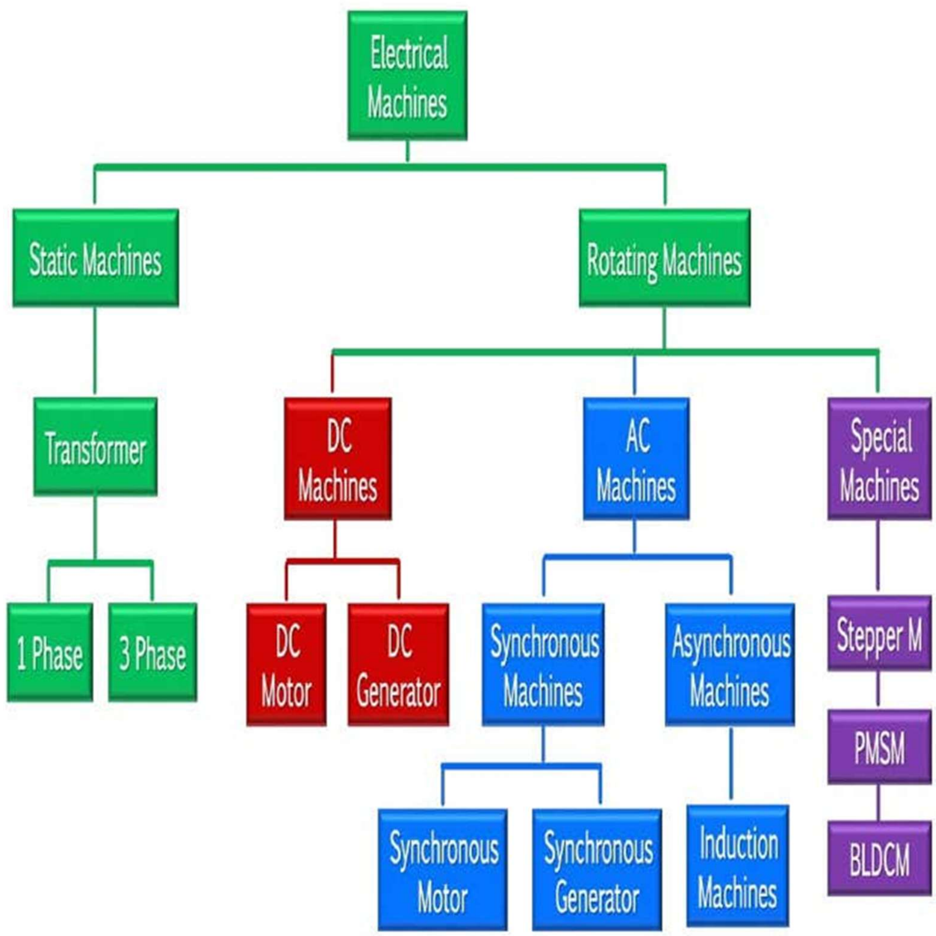

Fig.4.1 Types of Electrical Machines [Source: www.scribid.com]

4.3 Three Phase Synchronous Generators

4.3.1 Introduction

- Electric supply used now a days for commercial as well as domestic purposes is of alternating type.
- **The machines generating ac emf are called Alternators**. The alternators work at a specific constant speed called synchronous speed and hence in general called **Synchronous Generators**.
- The main difference between dc generators and alternators is that in alternators the field is rotating while armature is stationary and the commutator is absent.

4.3.2 Concept of Slip Rings and Brush Assembly

- In some alternators, the armature in which emf gets induced is rotating while the field to which dc supply is given is stationary.
- The load to which generated ac emf is to be supplied is always stationary.
- The arrangement which is used to collect an induced emf from the rotating armature and make it available to the stationary load circuit is called **Slip Ring and Brush Assembly**.

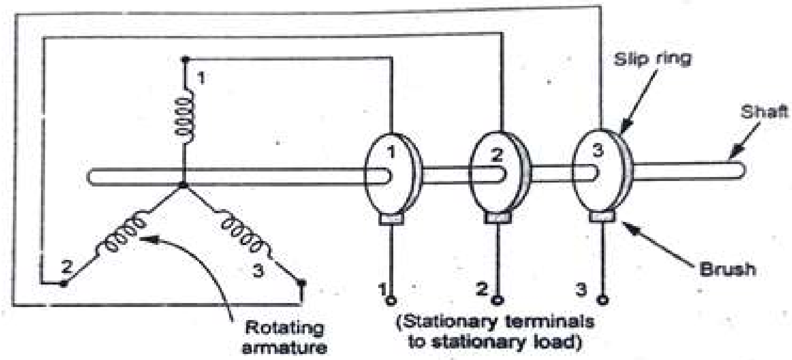

Fig.4.2 Arrangement of slip rings

- The armature winding is three phase and connected in generally in star. The three end terminals of armature winding are brought out.
- The slip rings, made up of conducting material are mounted on the shaft. Each terminal of armature winding is connected to an individual slip ring, permanently.
- Hence three phase supply generated in the armature is now available across the rotating slip rings.
- The brushes are resting on the slip rings, just making contact and are stationary.
- Hence any stationary load can then be connected across these stationary terminals available from the brushes.

The schematic arrangement is as shown in the Fig.4.2

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4.3.3 Construction

- In alternators the stationary winding is called **'Stator'** while the rotating winding is called **'Rotor'**.
- Most of the alternators have stator as armature and rotor as field, in practice.

4.3.4.1 Stator

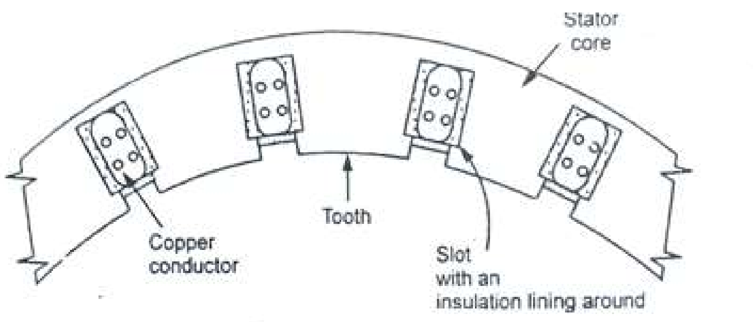


Fig.4.3. Section of an alternator stator

- ► The stator is a stationary armature.
- ► This consists of a core and slots to hold the armature winding similar to the three phase induction motor.
- ► The stator core uses a laminated construction.
- ► It is built up of special steel stampings insulated from each other with varnish or paper. The laminated construction is basically to keep down eddy current losses.
- ► Generally material used is steel to keep down hysteresis losses.
- ► The entire core is fabricated in a frame made of steel plates. The core has slots on its periphery for housing the armature conductors. Frame does not carry any flux and serves as the support to the core.
- ► Ventilation is provided with the help of holes casted in the frame.

4.3.4.2 Rotor

► There are two types of rotors used in alternators,

1. Salient Pole or Projected Pole Type
2. Smooth Cylindrical or Non Salient Type

4.3.4 Salient Pole Type

4.3.4.1 Rotor

- This is also called **projected pole type** as all the poles are projected out from the surface of the rotor.
- The poles are built up of thick steel laminations
- The poles are bolted to the rotor as shown in the Fig.4.4.
- The pole face (pole shoe) has been given a specific shape.
- The field winding is provided on the pole shoe.
- These rotors have large diameters and small axial lengths.
- As mechanical strength of salient pole type is less, this is preferred for low speed alternators ranging from 125 rpm to 500 rpm.
- The prime movers used to drive such rotor are generally water turbines and IC engines.

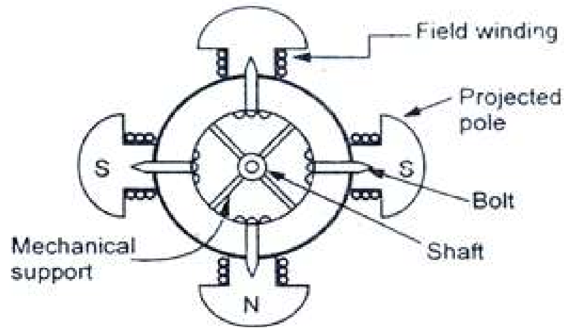


Fig.4.4 Salient pole type rotor

4.3.5.2 Smooth Cylindrical Type Rotor

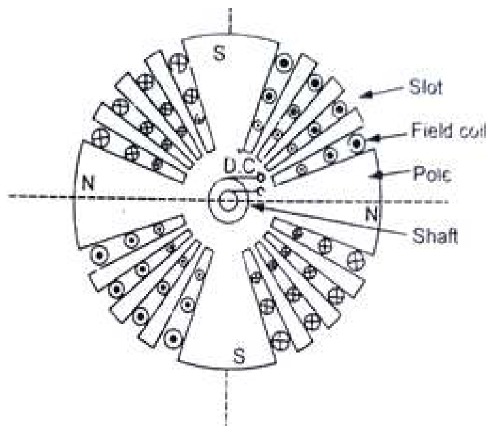


Fig.4.5. Non Salient pole type rotor

- This is also called **non salient type** or **non-projected pole type of rotor**. The Fig.4.5. shows smooth cylindrical Type of rotor.
- The rotor consists of smooth solid steel cylinder, having number of slots to accommodate the field coil.
- The slots are covered at the top with the help of steel or manganese wedges.
- The un-slotted portions of the cylinder itself act as the poles.
- The poles are not projecting out and the surface of the rotor is smooth which maintains uniform air-gap between stator and the rotor.
- These rotors have small diameters and large axial lengths. This is to keep peripheral speed within limits.
- These are mechanically very strong and thus preferred for high speed alternators ranging between 1500 to 3000 rpm.
- Such high-speed alternators are called Turbo-alternators.
- The prime movers used to drive such type of rotors are generally steam turbines, electric motors.

4.3.6 Difference between Salient pole and Cylindrical Type of Rotor

Sl. No	Salient or Projected Pole Type	Smooth Cylindrical or Non-salient Type
1	Poles are projecting out from the surface.	Un-slotted portion of the cylinder acts as poles hence poles are non-projecting.
2	Air gap is non uniform.	Air gap is uniform due to smooth cylindrical periphery.
3	Diameter is high and axial length is small.	Small diameter and large axial length.
4	Mechanically weak	Mechanically robust.
5	Preferred for low speed alternators.	Preferred for high speed alternators.
6	Prime movers used are water turbines , IC engines	Prime movers used are steam turbines, electric motors.
7	For same size the rating is smaller than cylindrical type.	For same size the rating is higher than salient pole type.
8	Separate damper winding is provided.	Separate damper winding is not necessary.

4.3.7 Working principle of an Alternator

The alternators work on the principle of **Electromagnetic Induction**.
When there is relative motion between the conductors and the flux, emf gets induced in the conductors. Though in an alternator the conductors are stationery and field is rotating, for understanding purpose we can always consider relative motion of conductors with respect to the flux produced by the field winding.
Consider a relative motion of a single conductor under the magnetic field produced by two stationary poles. The magnetic axis of the two poles produced by field is vertical shown dotted in the Fig.4.6.

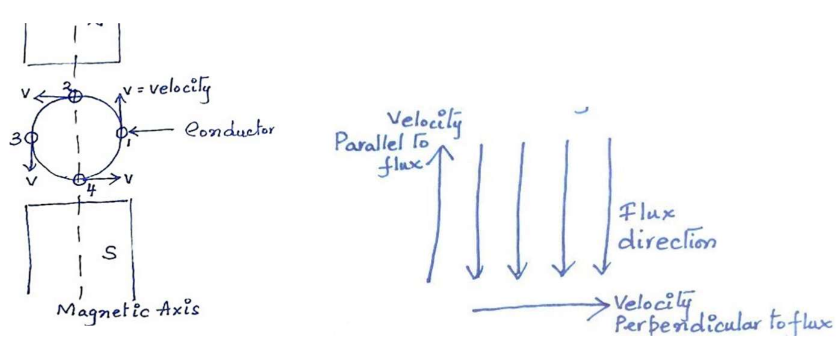


Fig. 4.6. Two pole alternator

Let conductor starts rotating from position 1. At this instant, the entire velocity component is parallel to the flux lines. Hence there is no cutting of flux lines by the conductor. Hence induced emf in the conductor is also zero.

As the conductor moves from the position 1 towards position 2, the part of the velocity component becomes perpendicular to the flux lines and proportional to that, emf gets induced in the conductor. The magnitude of such an induced emf increases as the conductor moves from position 1 towards 2.

At position 2, the entire velocity component is perpendicular to the flux lines. Hence there exists maximum cutting of the flux lines.

And at this instant, the emf induced in the conductor is at its maximum. As the position of the conductor changes from 2 towards 3, the velocity component perpendicular to the flux starts decreasing and hence induced emf magnitude also starts decreasing. At position 3, again the entire velocity component is parallel to the flux lines and hence at this instant induced emf in the conductor is zero.

As the conductor moves from position 3 towards 4, the velocity component perpendicular to the flux lines again starts increasing. But the direction of velocity component now is opposite to the direction of velocity component existing during the movement of the conductor from position 1 to 2.

Hence an induced emf in the conductor increases but in the opposite direction.

At position 4, it achieves maxima in the opposite direction, as the entire velocity component becomes perpendicular to the flux lines.

Again from position 4 to 1, induced emf decreases and finally at position 1, again becomes zero. This cycle continues as conductor rotates at a certain speed.

So, if we plot the magnitudes of the induced emf against the time, we get an alternating nature of the induced emf as shown in the Fig.4.7.

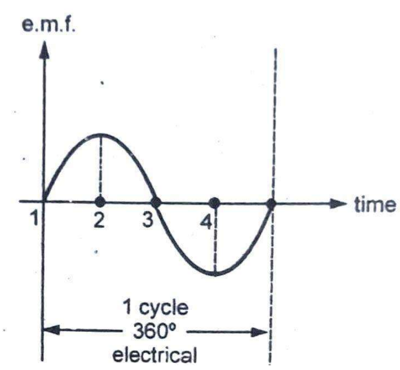


Fig. 4.7. Alternating nature of the induced emf | e.m.f.

Thus for 2 pole alternator, one mechanical revolution corresponds to one electrical cycle i.e. 360° electrical of an induced emf.

Mechanical and Electrical Angle

Consider 4 pole alternator i.e. the field winding is designed to produce 4 poles.

Due to 4 poles, the magnetic axis exists diagonally shown dotted in the Fig.4.8.

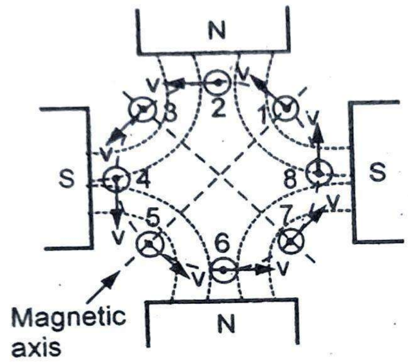


Fig. 4.8. 4-pole alternator

Now in position 1 of the conductor, the velocity component is parallel to the flux lines while in position 2, there is gathering of flux lines and entire velocity component is perpendicular to the flux lines. So, at position 1, the induced emf in the conductor is zero while at position 2, it is maximum.

Similarly, as conductor rotates, the induced emf will be maximum at position 4,6 and 8 and will be zero at position 3,5 and 7.

So, during one complete revolution of the conductor, induced emf will experience four times maxima, twice in either direction and four times zero. This is because of the distribution of flux lines due to existence of four poles. So, if we plot nature of the induced emf for one revolution of the conductor we get the two electrical cycles of the induced emf as shown in the Fig 4.9.

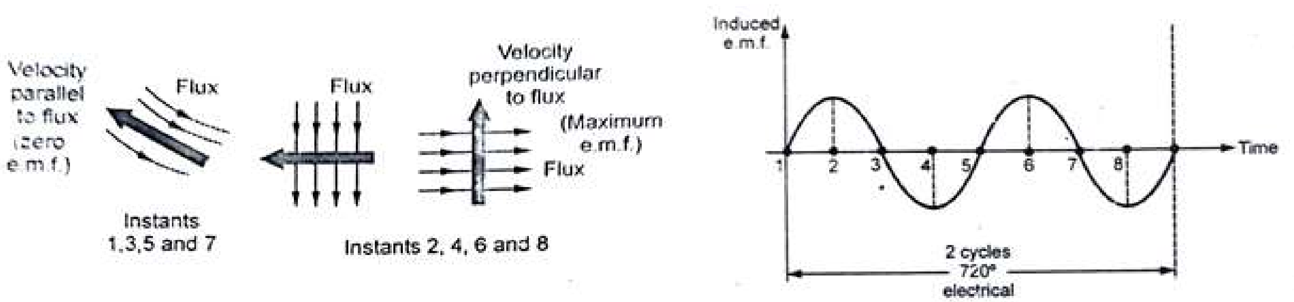


Fig.4.9. Nature of the induced emf

Thus, the degrees electrical of the induced emf i.e., number of cycles of the induced emf depends on the number of poles of an alternator.

So for a four pole alternator we can write,

$$360^\circ \text{ mechanical} = 720^\circ \text{ electrical}$$

From this we can establish the general relation between degrees mechanical and degrees electrical as,

$$360° \text{ mechanical} = 360° \times \frac{P}{2} \text{ electrical}$$

Where P = Number of Poles

i.e. $$\mathbf{1° \text{ Mechanical} = \left[\frac{P}{2}\right]^0 \text{ electrical}}$$

Frequency of Induced EMF

Let P = Number of poles

N = speed of the rotor in rpm

F= frequency of the induced emf

One mechanical revolution of rotor = P/2 cycles of emf electrically

Thus there are P /2 electrical cycles per revolution.

As speed N rpm in one second rotor will complete $\left(\frac{N}{60}\right)$ revolutions.

But electrical cycles / second = Frequency = f

∴ Frequency f = (No. of electrical cycles per revolution) × (No. of revolutions per second)

$$\mathbf{f} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \textbf{ Hz}$$

So there exists a fixed relation between three quantities. The number of poles P, the speed of the rotor N in rpm and f the frequency of an induced emf in Hz.

Synchronous Speed (N_S)

- For fixed number of poles (P), alternator has to be rotated at a particular speed to keep the frequency of the generated emf constant at the required value. Such a speed is called synchronous speed of the alternator denoted as N_s.
- $\mathbf{N_S} = \frac{120f}{P}$
- Where f = Required rated frequency.
- In our nation, the frequency of an alternating emf standard equal to 50 Hz.
- To get 50Hz frequency, for different number of poles, alternator must be driven at different speeds called synchronous speeds.
- Following table gives the values of the synchronous speeds for the alternators having different number of poles.

Number of Poles	**Synchronous Speed N_s in r.p.m**
2	3000
4	1500
8	750

Number of Poles	**Synchronous Speed N_s in r.p.m**
12	500
24	250

From the table it can be seen that minimum number of poles for an alternator can be two hence maximum value of synchronous speed possible in our nation i.e., for frequency of 50Hz is 3000 rpm.

Such a machine bearing a fixed relationship between P, N and f is called **synchronous machine** and hence alternators are also called **synchronous generators**.

EMF Equation of An Alternator

Let φ = Flux per pole, in Wb

P = Number of poles

N_S = Synchronous speed in rpm

f = frequency of induced emf in Hz

Z = total number of conductors.

Z_{ph} = Conductors per phase = $\frac{Z}{3}$ as number of phases = 3

Consider a single conductor placed in a slot.

The average value of emf induced in a conductor = $\frac{d\varphi}{dt}$

For one revolution of a conductor,

$$\mathbf{e_{avg}\ per\ conductor} = \frac{Flux\ cut\ in\ one\ revolution}{Time\ taken\ for\ one\ revolution}$$

The total flux cut in one revolution is φ×P

Time taken for one revolution is $\frac{60}{N_S}$ seconds

$$\therefore \mathbf{e_{avg}\ per\ conductor} = \frac{\varphi P}{\left(\frac{60}{N_S}\right)} = \frac{\varphi P N_S}{60}$$

But f = $\frac{PN_S}{120}$ i.e $\frac{PN_S}{60} = 2f$

Substituting in above equation,

e_{avg} per conductor =2fφ volts

Assume full pitch winding for simplicity i.e this conductor is connected to a conductor which is 180° electrical apart.

So these two emfs will try to set up a current in the same i.e the two emfs are helping each other and hence resultant emf per turn will be twice the emf induced in a conductor.

$\therefore$ **emf per turn = 2 × (emf per conductor)**

= 2 × (2fφ) = 4fφ volts

Let T_{ph} be the total number of turns per phase connected in series.

$\therefore$ Average $E_{ph} = T_{ph} \times$ (Average emf per turn)

$= T_{ph} \times 4f\varphi$

But in ac circuits RMS value of an alternating quantity is used for the analysis. The form factor is 1.11 of sinusoidal emf.

$$K_f = \frac{RMS}{Average} = 1.11 \quad \text{.........For sinusoidal}$$

$\therefore$ RMS value of $E_{ph} = K_f \times$ Average value

$= 1.11 \times 4f\varphi T_{ph}$

$\therefore$ **RMS value of $E_{ph} = 4.44\ f\ \varphi\ T_{ph}$ volts**

This is the basic emf equation for an induced emf per phase for full pitch, concentrated type of winding where T_{ph} = Number of turns per phase

$$T_{ph} = \frac{Z_{ph}}{2} \quad \text{........As 2 conductors constitute 1 turn}$$

But, as mentioned earlier, the winding used for the alternators is distributed and short pitch hence emf induced slightly gets affected. This effect is considered by the factors called **winding factors**.

1) **Pitch Factor or Coil Span Factor (K_c)**
2) **Distribution Factor (K_d)**

So generalized expression for emf equation considering the winding factors is given by,

$$E_{ph} = 4.44\ K_C\ K_d\ f\ \varphi\ T_{ph} \text{ volts}$$

For full pitch coil, $K_C = 1$ and for concentrated winding $K_d = 1$

For a star connected alternator the terminal voltage is $\sqrt{3}$ times the phase voltage hence,

$$E_{line} = \sqrt{3} \times E_{ph} = \sqrt{3} \times 4.44\ K_C\ K_d\ f\ \varphi\ T_{ph} \text{ volts (For star connected)}$$

The product of K_d and K_C is called winding factor denoted as K_w. Thus $K_w = K_C \times K_d$.

Numerical 1

A 3φ, 4 pole, 50 Hz star connected alternator has 36 slots and 30 conductors per slot. The useful flux per pole is 0.05Wb. Find synchronous speed and line voltage on no load. Assume winding factor 0.96.

P = 4, f = 50Hz, 36 slots and 30 conductors per slot, φ = 0.05 Wb, K_d = 0.96, Assume full pitch winding, $K_c = 1$

- $N_S = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$ rpm
- Z = Total number of conductors = Slots × conductors / slot = 36 × 30 = 1080
- $Z_{ph} = \frac{Z}{3} = \frac{1080}{3} = 360$
- $T_{ph} = \frac{Z_{ph}}{2} = \frac{360}{2} = 180$ 2 conductor = 1 turn

$\therefore$ $E_{ph} = 4.44\ K_C\ K_d\ f\ \varphi\ T_{ph}$

$= 4.44 \times 1 \times 0.96 \times 0.05 \times 50 \times 180$

$E_{ph} = 1918.08$ V

$E_{line} = \sqrt{3} \times E_{ph} = 3.322$kV

Numerical 2

A 2 pole, 3 phase alternator running at 3000 rpm has 42 armature slots with 2 conductors in each slot. Calculate the flux/pole required to generate a line voltage of 2300V. Distribution factor is 0.952 and the pitch factor is 0.956

P = 2, N_S = 3000 rpm, Eline = 2300V, Kd = 0.952 , K_c = 0.956 , 42 armature slots with 2 conductors in each slot

- $N_S = \frac{120f}{P}$ i.e $f = \frac{2 \times 3000}{120} = 50$Hz
- $E_{ph} = \frac{E_{line}}{\sqrt{3}} = \frac{2300}{\sqrt{3}} = 1327.9056$ Assuming star
- Z = Total number of conductors = Slots × conductors / slot = 42 × 2 = 84
- $Z_{ph} = \frac{Z}{3} = \frac{84}{3} = 28$
- $T_{ph} = \frac{Z_{ph}}{2} = \frac{28}{2} = 14$ 2 conductor = 1 turn

$\therefore$ $E_{ph} = 4.44\ K_C\ K_d\ f\ \varphi\ T_{ph}$

$1327.9056 = 4.44 \times \varphi \times 50 \times 0.952 \times 0.956 \times 14$

$\varphi = 0.4694$ Wb

4.4 AC Motors

4.4.1 Introduction

An electric motor which operates on a.c. supply is called ac motor. As a.c. supply is commonly available, the a.c. motors are very popularly used in practice.

► The ac motors are classified as:

1. Single And Three Phase Induction Motors
2. Synchronous Motors
3. Some Special Purpose Motors

Out of all these types Three phase induction motors are widely used for various industrial applications.

4.4.2 Three Phase Induction Motor

4.4.2.1 Rotating Magnetic Field

In case of a 3-phase system, there exists 3 windings which are displaced by an angle of 120^0 .

The current flowing in each phase will produce respective fluxes in the direction shown below

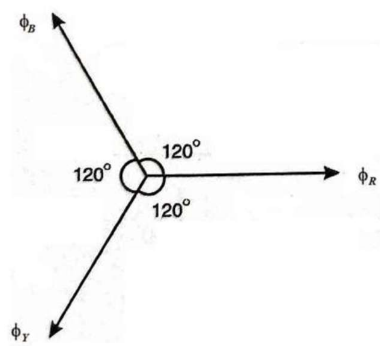


Fig. 4.10 Rotating Magnetic Field

The fluxes are represented by the following equation

$$\Phi_R = \Phi_m \sin \omega t = \Phi_m \sin \theta$$
$$\Phi_Y = \Phi_m \sin(\omega t - 120°) = \Phi_m \sin(\theta - 120°)$$
$$\Phi_B = \Phi_m \sin(\omega t - 240°) = \Phi_m \sin(\theta - 240°)$$

The resultant of the three alternating fluxes, separated from each other by 120° has constant amplitude of $1.5\varphi_m$ where φ_m is the maximum amplitude of an individual flux due to any flux.

The resultant flux always keeps on rotating with a certain speed in space.

There exists a fixed relation between frequency f of ac supply to the windings, the number of poles P for which winding is wound and speed N rpm of rotating magnetic field.

For a standard frequency whatever speed of RMF results is called synchronous speed, in case of induction motors.

It is denoted as N_s.

$$\mathbf{N_S} = \frac{120}{P} = \text{speed of RMF}$$

Where f = supply frequency in Hz

P = Number of poles for which winding is wound.

The direction of rotating magnetic field depends on the phase sequence of the three phase supply.

By interchanging any two terminals of three phase winding while connecting it to three phase ac supply, direction of rotation of RMF gets reversed.

Thus, by changing the supply phase sequence, the direction of three phase induction motor can be reversed.

4.4.2.2 Construction

► Basically the induction motor consists of two main parts, namely

1. The part i.e three phase windings which is stationary called stator.
2. The part which rotates and is conected to the mechanical load through shaft called rotor.

The conversion of electrical power to mechanical power takes place in rotor. Hence rotor develops a driving torque and rotates.

<u>Stator:</u>

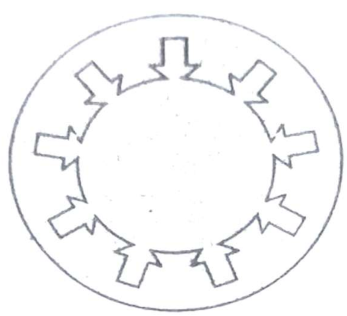


Fig. 4.11 Stator front view

- The stator has laminated type of construction made up of stampings which are 0.4 to 0.5mm thick.
- The stampings are slotted on its periphery to carry the stator windings.
- The stampings are insulated from each other.Such a construction essentially keeps the iron losses to a minimum value.
- The number of stampings are stamped together to build the stator core.
- The built up core is then fitted in a casted or fabricated steel frame.
- The choice of material for the stamping is generally silicon steel, which minimises the hysteresis loss.
- The slots on the periphery of the stator core carries a three phase winding , connected either in star or delta. This three phase winding is called stator winding. It is wound for definite number of poles.
- The radial ducts are provided for the cooling purposes. (Fig, 4.11 and 4.12)

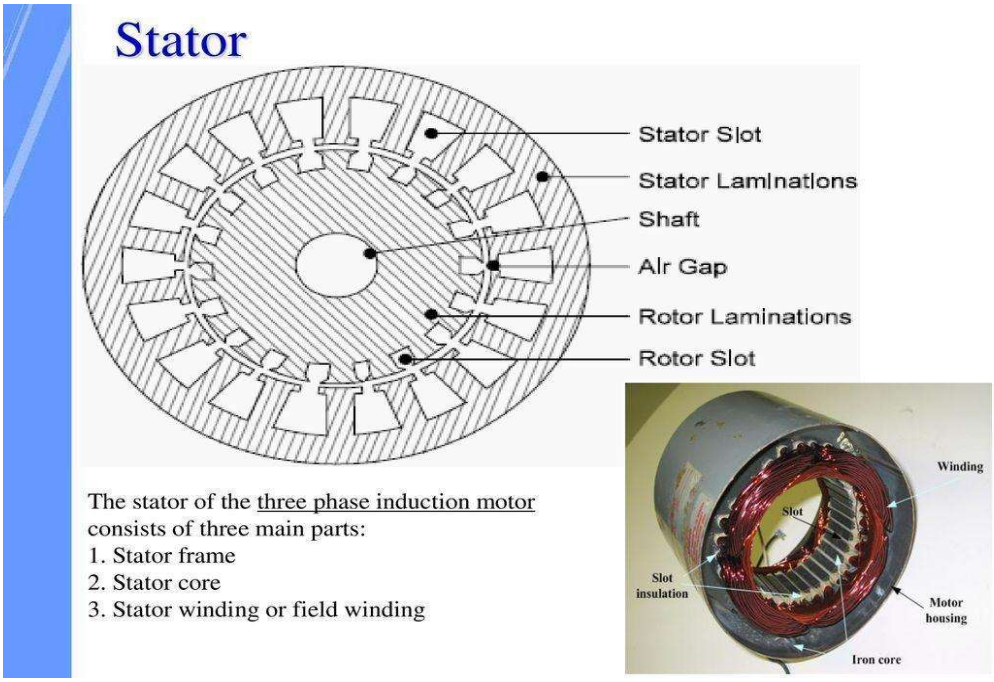


Fig.4.12 Stator 3D View

Rotor

The rotor is placed inside the stator .

The air gap between stator and the rotor is 0.4mm to 4mm.

► The two types of rotor constructions which are used for induction motors are

1. Squirrel cage rotor
2. Slip ring or phase wound rotor

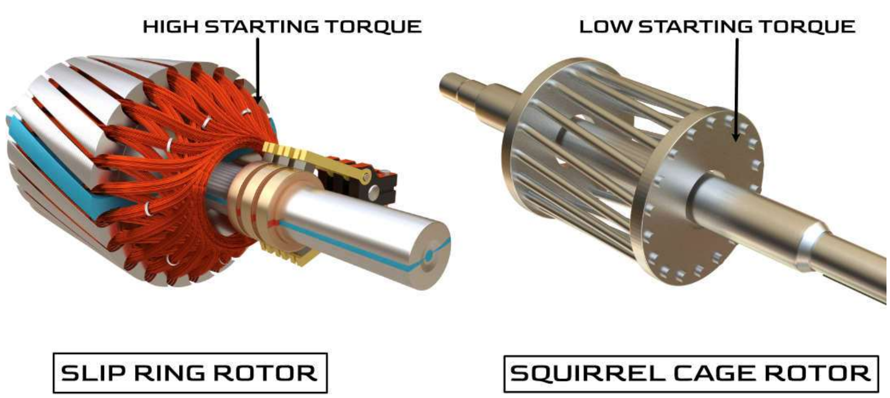


Fig. 4.13 Different types of Rotors

a. Squirrel Cage Rotor

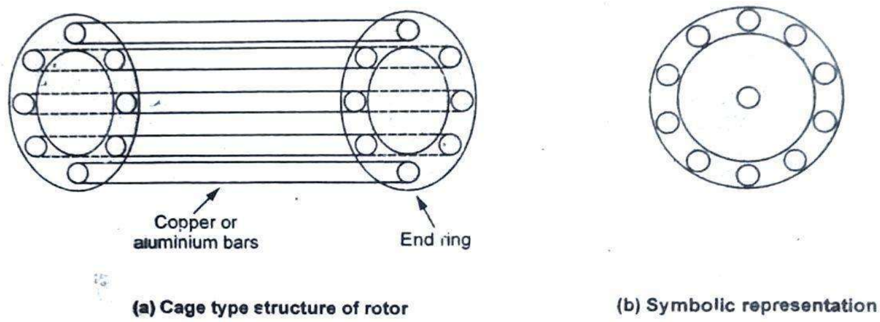


Fig.4.14 Squirrel Cage Rotor

- ► The rotor core is cylindrical and slotted on its periphery.
- ► The rotor consists of un-insulated copper or aluminium bars called rotor conductors.
- ► The bars are placed in the slots.
- ► These bars are permanently shorted at each end with the help of conducting copper ring called end ring. The bars are usually brazed to the end rings to provide good mechanical strength. The entire structure looks like a cage, forming a closed electrical circuit. So the rotor is called squirrel cage rotor. The construction is shown in the Fig
- ► As the bars are permanently shorted to each other through end ring, the entire rotor resistance is very very small. Hence this rotor is also known as short circuited rotor.
- ► As rotor itself is short circuited , no external resistance can have any effect on the rotor resistance . hence no external resistance can be introduced in the rotor circuit. So slip ring and brush assembly is not required for this rotor.Hence the construction of this rotor is very simple.
- ► Fan blades are provided at the ends of the rotor core.This circulates the air through the machine while in operation, providing the necessary cooling.
- ► In this type of rotor, the slots are not arranged parallel to the shaft axis but are skewed as shown in the Fig. 4.15

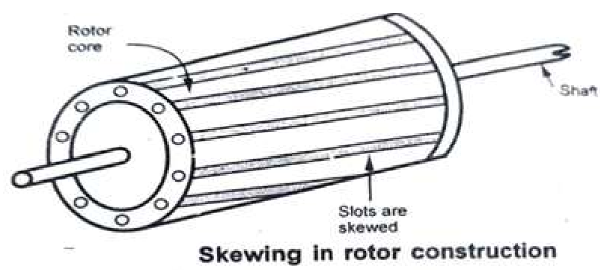


Fig. 4.15 Squirrel cage rotor skew construction

Advantages of Skewing

- ► The advantages of skewing are,

1. A magnetic hum i.e noise gets reduced due to skewing.Hence skewing makes the motor operation quiter.
2. It makes the motor operation smooth.
3. The stator and rotor teeth may get magnetically locked. Such a Tendency of magnetic locking gets reduced due to skewing.
4. It increases the effective transformation ratio between stator and rotor.

b. Slip Ring Rotor or Phase Wound Rotor

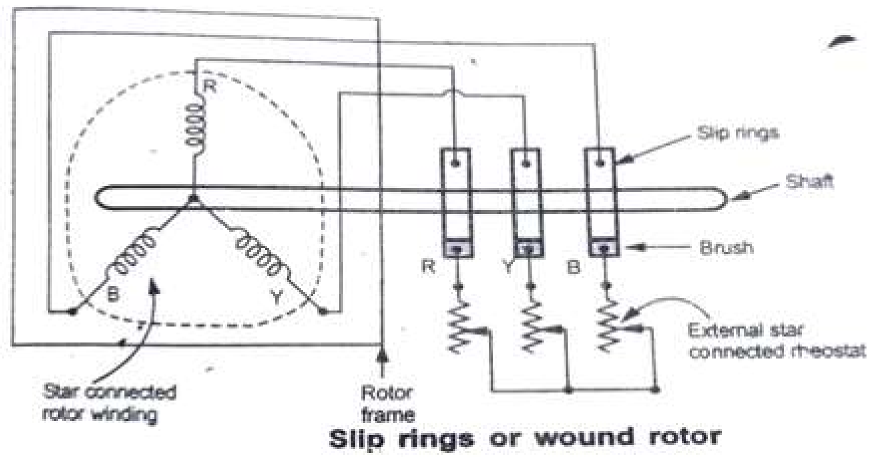


Fig.4.16 Phase wound rotor

- In this type of construction rotor winding is exactly similar to the stator. (Fig. 4.16)
- The rotor carries a three phase star or delta connected, distributed winding, wound for same number of poles as that of stator.
- The rotor construction is laminated and slotted. The slots contain the rotor winding.
- The three ends of three phase winding, available after connecting the winding in star or delta, are permanently connected to the slip rings.
- With the help of slip rings, the external resistance can be added in series with each phase of the rotor winding.This arrangement is shown in the Fig.
- This way the value of the rotor resistance per phase can be controlled. This helps us to control some of the important characteristics of the motor like starting torque , speed etc.
- In the running condition, the slip rings are shorted.
- The possibility of addition of an external resistance in series with the rotor, with the help of slip rings is the main feature of this type of rotor.

4.4.2.3 Comparison of Squirrel Cage and Wound Rotor

Sl . No	Wound Rotor	Squirrel Cage Rotor
1	Rotor consists of three phase winding similar to the stator winding.	Rotors consists of bars which are shorted at the ends with the help of end rings.
2	Construction is complicated.	Construction is very simple.
3	Resistance can be added externally.	As permanently shorted external resistance can not be added.
4	Slip rings and brushes are present.	Slip rings and brushes are present.
5	The construction is delicate and due to the brushes , frequent maintenance is necessary.	The construction is robust and maintenance free.
6	The rotors are very costly	Due to simple construction rotors are cheap.

Sl . No	Wound Rotor	Squirrel Cage Rotor
7	Only 5% of induction motors in industry use slip ring rotor.	Very common and almost 95% induction motors use this type of rotor.
8	High starting torque can be obtained.	Moderate starting torque which cannot be controlled.
9	Rotor resistance starter can be used.	Rotor resistance starter cannot be used.
10	Rotor must be wound for the same number of poles as that of stator.	The rotor automatically adjusts itself for the same number of poles as that of stator.
11	Speed control by rotor resistance is possible.	Speed control by rotor resistance is not possible.
12	Rotor copper losses are high hence efficiency is less.	Rotor copper losses are less hence have higher efficiency.
13	Used for lifts, hoists, cranes, elevators, compressors etc.	Used for lathes, drilling machines, fans, blowers, water pumps, grinders, printing machines etc.

4.4.2.4 Principle of Operation of Induction motor

- Induction motors works on the principle of electromagnetic induction.
- When three phase supply is given to the three phase stator winding, a rotating magnetic field of constant magnitude is produced. The speed of this rotating magnetic field is synchronous speed, N_S rpm.
- $N_S = \frac{120f}{P}$ = speed of RMF
- This rotating field produces an effect of rotating poles around a rotor.
- Let direction of rotation of this rotating magnetic field is clockwise as shown in the Fig. 4.17

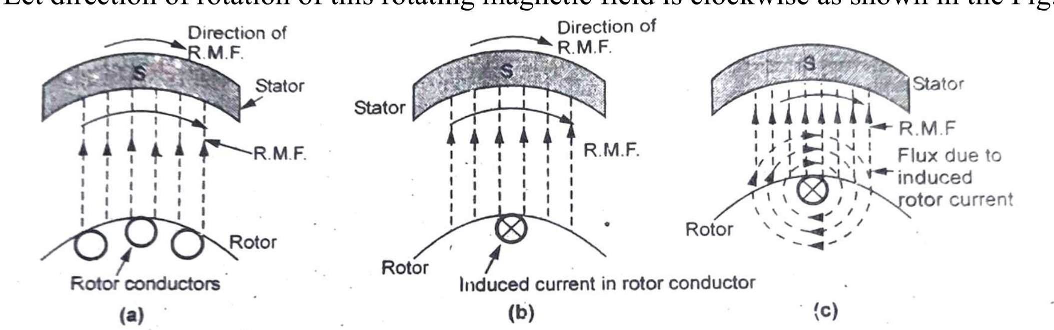


Fig. 4.17 RMF in induction motor

- Now at this instant the rotor is stationary and stator flux RMF is rotating. So its obvious that there exists a relative motion between the RMF and rotor conductors.
- Whenever conductor cuts the flux, emf gets induced in it. So the emf gets induced in the rotor conductors called rotor induced emf. This is electro magnetic induction.
- As rotor forms closed circuit , induced emf circulates current through rotor called rotor current as shown in the fig. 4.17b. Let the direction of this rotor current is going into the paper denoted by a cross as shown in the fig. 4.17c

- ► Any current carrying conductor produces its own flux. So rotor produces its flux called rotor flux. For assumed direction of rotor current , the direction of rotor flux is clockwise as shown in fig. 4.17c
- ► Both the fluxes interact with each other as shown in Fig. 4.18

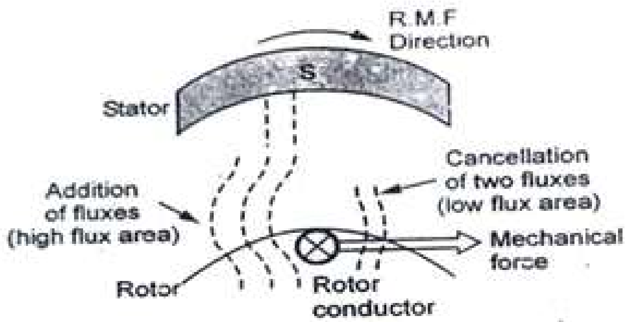


Fig. 4.18 Interaction of fluxes

On left of rotor conductor, two fluxes are in same direction hence add up to get high flux area.

- On right side , two fluxes cancel each other to produce low flux area.
- As flux lines act as stretched rubber band , high flux density area exerts a push on rotor conductor towards low flux density area. So rotor conductor experiences a force from left to right in this case as shown in the Fig. 4.18
- As all the rotor conductors experience a force , the overall rotor experiences a torque and starts rotating.
- So interaction of the two fluxes is very essential for a motoring action.
- According to Lenz's Law the direction of induced current in the rotor is so as to oppose the cause producing it.
- The cause of rotor current is the induced emf which is induced because of relative motion present between the rotating magnetic field and the rotor conductors.
- Hence to oppose the relative motion i.e to reduce the relative speed, the rotor experiences a torque in the same direction as that of RMF and tries to catch up the speed of rotating magnetic field.

N_s = Speed of rotating magnetic field in rpm

N = Speed of rotor i.e motor in rpm

$N_s - N$ = Relative speed between the two, rotating magnetic field and the rotor

Thus rotor always rotates in same direction as that of RMF.

Can $N = N_s$?

When rotor starts rotating, it tries to catch the speed of rotating magnetic field . If it catches the speed of the RMF , the relative motion between rotor and the rotating magnetic field will vanish ($N_s - N = 0$)

Infact the relative motion is the main cause for the induced emf in the rotor. So induced emf will vanish and hence there cannot be rotor current and the rotor flux which is essential to produce the torque on the rotor.

Eventually motor will stop. But immediately there will exist a relative motion between rotor and rotating magnetic field and it will start.

But due to inertia of rotor , this does not happen in practice and rotor continues to rotate with a speed slightly less than the synchronous speed of the RMF in the steady state.

The induction motor never rotates at synchronous speed. The speed at which it rotates is hence called subsynchronous speed and motor sometimes called asynchronous motor.

$\therefore N < N_S$

So it can be said that rotor slips behind the rotating magnetic field produced by stator.

The difference between the two is called slip speed of the motor.

N_S – N = Slip speed of the motor in rpm

This speed decides the magnitude of the induced emf and the rotor current, which in turn decides the torque produced.

4.4.2.5 Slip of Induction Motor

The slip speed (N_s – N) is generally expressed as the percentage of the synchronous speed.

Slip of the induction motor is defined as the difference between the synchronous speed (N_s) and the actual speed of rotor i.e motor (N) expressed as a fraction of the synchronous speed (N_s). This is also called absolute slip or fractional slip and is denoted as 's'.

Thus, $s = \frac{N_S - N}{N_S}$

The percentage slip is expressed as

$$\% \, s = \frac{N_S - N}{N_S} \times 100$$

In terms of slip, the actual speed of motor (N) can be expressed as,

$N = N_s (1 - s)$

At start, motor is at rest and hence its speed N is zero.

$\therefore \quad s = 1$ at start

This is maximum value of slip s possible for induction motor which occurs at start.

While s = 0 gives us $N = N_S$ which is not possible for an induction motor. So slip of an induction motor cannot be zero under any circumstances.

Practically motor operates in the slip range of 0.01 to 0.05 i.e 1% to 5%. The slip corresponding to full load speed of the motor is called full load slip.

Effect of Slip on The Rotor Frequency

In case of induction motor, the speed of rotating magnetic field is $N_S = \frac{120f}{P}$

At start when N = 0, s=1 and stationary rotor has maximum relative motion with respect to RMF. Hence maximum emf gets induced in the rotor at start.

The frequency of this induced emf at start is same as that of supply frequency.

As motor actually rotates with speed N, the relative speed of rotor with respect to RMF decreases and becomes equal to slip speed of N_S – N.

The induced emf in rotor depends on rate of cutting of flux i.e relative speed $N_S - N$.

Hence in running condition magnitude of induced emf decreases so as its frequency.

The rotor is wound for same number of poles as that of stator i.e P.

If f_r is the frequency of rotor induced emf and rotor currents, in running condition at slip speed $N_S - N$ then there exists a fixed relation between $(N_S - N)$, f_r and P similar to equation

So, we can write for rotor in running condition,

$$(N_S - N) = \frac{120 f_r}{P}$$

Rotor poles = Stator poles = P

$$\frac{N_S - N}{N_S} = \frac{\frac{120 f_r}{P}}{\frac{120 f}{P}} \quad \text{but} \quad \frac{N_S - N}{N_S} = \text{Slip } s$$

i.e $S = \dfrac{f_r}{f}$

$$\mathbf{f_r = s\, f}$$

As slip of the induction motor is in the range 0.01 to 0.05 rotor frequency is very small in the running condition.

4.4.2.6 Applications

- **Squirrel Cage Type Induction Motors:**

  Squirrel Cage Type of motors having moderate starting torque and constant speed characteristics preferred for driving fans, Blowers, Water Pumps, Grinders ,Lathe Machines, Printing Machines, Drilling Machine.

- **Slip Ring Induction Motors:**

  Slip Ring Induction Motors can have high starting torque as high as maximum torque.

  Hence they are preferred for Lifts , Hoists, Elevators, Cranes, Compressors.

NUMERICALS ON INDUCTION MOTORS

1. A 4 pole 50 Hz induction motor is running at 1300 rpm. Find the speed of stator magnetic field with respect to the rotor?
   Solution:
   P=no. of poles=4
   Ns =Synchronous speed= 120f/p=120 x 50 / 4 =1500 rpm
   Actual speed= N= 1300 rpm
   Hence speed of stator magnetic field with respect to rotor is
   Ns – N =1500-1300 = 200 rpm

2. A 4 pole, 3 phase, 50 Hz induction motor runs at a speed of 1470 r.p.m. speed. Find the frequency of the induced e.m.f. in the rotor under this condition.
   Solution:
   P=no. of poles=4
   Supply frequency = f= 50Hz
   Ns = Synchronous speed=120f/P = 120 x 50/4 = 1500 RPM,
   N=Actual speed=1470 rpm
   Slip of an induction motor,
   s = (Ns – N)/Ns = (1500 – 1470)/1500 = 0.02
   Now Rotor frequency fr = s * f = 0.02 x 50 = 1 Hz

3. A 6-pole induction motor is running from 50Hz supply. The emf in its rotor is of frequency 2.5 Hz. Find the speed of the motor
   Solution:
   Ns = (120 ∗ f)/P
   Ns = (120 × 50)/6 = 1000rpm
   Rotor frequency=fr=s * f
   Slip, s= fr/f = 2.5/50 = 0.05
   N = Ns (1 − s)
   N = 1000 (1 − 0. 05) = 950rpm

A three-phase, 6-pole, 50-Hz induction motor has a slip of 1 % at no load and 3 % at full load. Find (*a*) the synchronous speed, (*b*) the no-load speed, (*c*) the full-load speed, (*d*) the frequency of rotor-currents at standstill, and (*e*) the frequency of rotor-currents at full load.

Solution

(*a*) The synchronous speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = \mathbf{1000\ rpm}$

(*b*) The no-load speed, $N = N_s(1 - s) = 1000(1 - 0.01) = \mathbf{990\ rpm}$

(*c*) The full-load speed, $N = N_s(1 - s) = 1000(1 - 0.03) = \mathbf{970\ rpm}$

(*d*) At standstill, $s = 1$. Hence, the frequency of rotor-currents,

$$f_r = sf = 1 \times 50 = \mathbf{50\ Hz}$$

(*e*) At full load, $s = 0.03$. Therefore, the frequency of rotor-currents,

$$f_r = sf = 0.03 \times 50 = \mathbf{1.5\ Hz}$$

A three-phase, 6-pole induction motor runs at 960 rpm on full load. It is supplied from a 4-pole alternator running at 1500 rpm. Calculate the full-load slip of the motor.

Solution The frequency of generated emf by the alternator is

$$f = \frac{NP}{120} = \frac{1500 \times 4}{120} = 50 \text{ Hz}$$

Therefore, the synchronous speed of the motor is

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\therefore \quad \text{Slip, } s = \frac{N_s - N}{N_s} = \frac{1000 - 960}{1000} = 0.04 = \mathbf{4\ \%}$$

4.5 DC Generator

- All generators work on the principle of dynamically induced e.m.f.
- The principle is nothing but the faraday`s law of electromagnetic induction. It states that "***Whenever the number of magnetic lines of force i.e. flux linking with the conductor or a coil changes, an electromotive force is set up in that conductor or coil***."
- The magnitude of the induced e.m.f in a conductor is proportional to the rate of change of flux associated with the conductor. This is mathematically given by,

$$e \propto \frac{d\varphi}{dt}$$

- The relative motion can be achieved by rotating conductor with respect to flux or by rotating flux with respect to a conductor.
- So a voltage gets generated in a conductor, as long as there exists a relative motion between the conductor and the flux.
- Such an induced e.m.f which is due to physical movement of the coil or conductor with respect to flux or movement of flux with respect to coil or conductor is called **dynamically induced e.m.f**.
- To have a large voltage at the output, a number of conductors are connected together in a specified manner, to form a winding. This winding is called **armature winding** of a D.C. machine.
- The part on which this winding is kept is called armature of a D.C. machine.
- To have the rotation of conductors, the conductors placed in the armature are rotated with the help of external device. Such an external device is called as **prime movers**.
- The commonly used prime movers are diesel engine, steam engine, steam turbines etc.
- The necessary magnetic flux is produced by current carrying winding which is called field winding.
- The direction of induced e.m.f is obtained by using Fleming`s right hand rule.
- If the angle between the plane of rotation and plane of flux is θ as measured from the axis of the plane of flux then the induced e.m.f is given by

$$E = B\,l\,(v \sin\theta)$$

- Where v sinθ is the component of velocity which is perpendicular to the plane of flux and hence responsible for the induced e.m.f.

4.5.1 Constructional details of a D.C. Machine

Figure 1 shows the constructional details of DC machine.

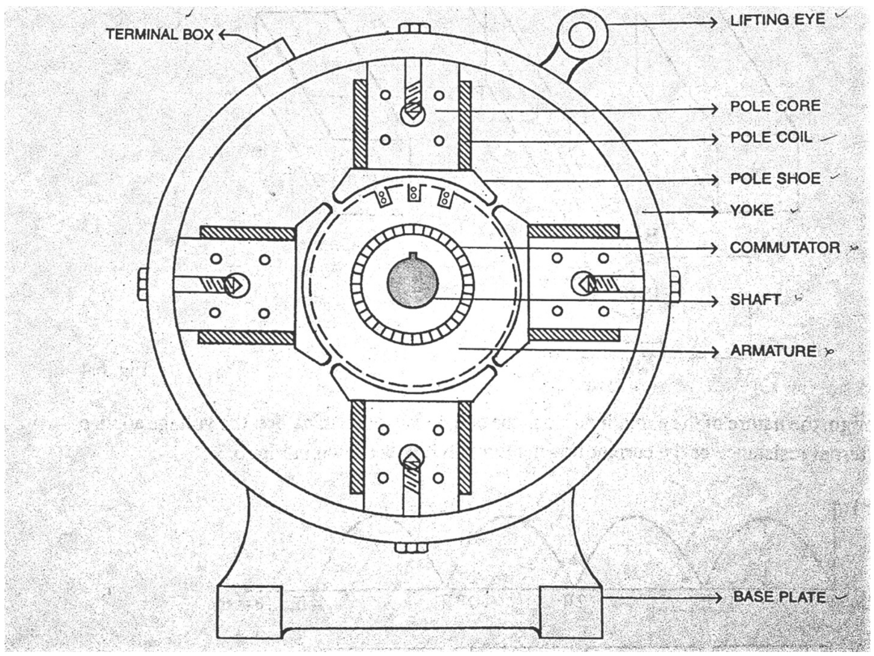


Fig. 1 Constructional details of D C Machine

DC machine consists of the following parts:

4.5.1.1 Yoke

a) Functions

- It serves the purpose of outermost cover of the D.C. machine. So that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases, acidic fumes etc.
- It provides mechanical support to the poles.
- It forms the part of the magnetic circuit. It provides the path for low reluctance for magnetic flux. The low reluctance path is important to avoid wastage of power to provide same flux. Large current and hence the power is necessary if the path has high reluctance, to produce the same flux.

b) Choice of material: It is prepared by using cast iron because it is cheapest and provides low reluctance path. For large machines rolled steel, cast steel, silicon steel is used which provides high permeability i.e. low reluctance and gives good mechanical strength.

4.5.1.2 Poles

- Each pole is divided into two parts namely, Pole core and Pole shoe
- Figure 3.2 shows the pole structure

a) Functions of pole core and pole shoe

- Pole core basically carries a field winding which is necessary to produce the flux.
- It directs the flux produced through air gap to armature core, to the next pole.
- Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced e.m.f. To achieve this, pole shoes has been given a particular shape.

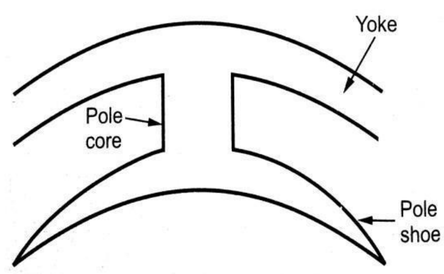


Fig. 2 Pole structure

b) Choice of material: It is made up of magnetic material like cast iron or cast steel. As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

4.5.1.3 Field Winding

- The field winding is wound on the pole core with a definite direction.

a) Functions

- The field winding carries current and behaves as an electromagnet, producing necessary flux**.**
- As it helps in producing magnetic field i.e. exciting the pole as an electromagnet it is called field winding or exciting winding.

b) Choice of material: It has to carry current hence obviously made up of some conducting material. So aluminium or copper is the choice. But field coils are required to take any type of shape and bend about pole core and copper has good pliability i.e. it can bend easily. So copper is the proper choice.

4.5.1.4 Armature

- The armature is further divided into two parts: Armature core and Armature winding

Armature core: Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

a) Functions

- Armature core provides house for armature winding i.e. armature conductors.
- To provide path of low reluctance to the magnetic flux produced by the field winding.

b) Choice of material

- As it has to provide low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.
- It is made up of laminated construction to keep eddy current loss as low as possible.

Armature winding: is nothing but the interconnection of armature conductors, placed in the slots provided on the armature core periphery. When the armature is rotated, in case of generator, magnetic flux gets cut by armature conductors and e.m.f gets induced in them.

a) Functions

- Generation of e.m.f takes place in the armature winding in case of generators.
- To carry the current supplied in case of D.C. motors.
- To do the useful work in the external circuit.

b) Choice of material: As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

4.5.1.5 Commutator

- The basic nature of e.m.f induced in the armature conductor is alternating.
- This needs rectification in case of D.C. generator, which is possible by a device called commutator.

a) Functions

- To facilitate the collection of current from the armature conductor.
- To convert internally developed alternating e.m.f to unidirectional e.m.f.
- To produce unidirectional torque in case of motors.

b) Choice of material:

- As it collects current from the armature, it is also made up of copper segments.
- It is cylindrical in shape and is made up of wedge shaped segments of hard drawn high conductivity copper. These segments are insulated from each other by thin layer of mica.

- Each commutator segment is connected to the armature conductor by means of copper strip.

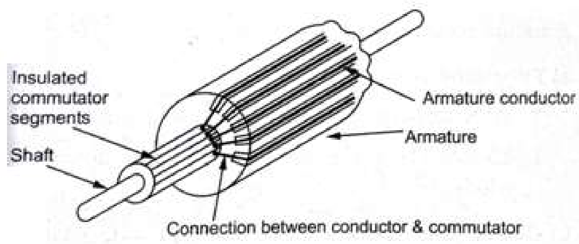


Fig 3 Commutator

4.5.1.6 Brushes and Brush gear

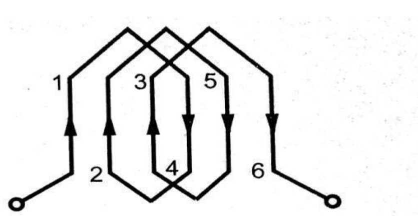
- Brushes are stationary and resting on the surface of the commutator.

a) Function: To collect current from commutator and make it available to the stationary external circuit.

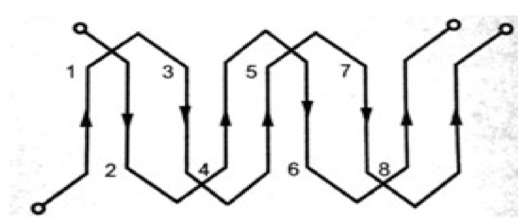
b) Choice of material: To avoid wear and tear of commutator brushes are normally made up of soft material like carbon.

4.5.2 Types of Armature Winding

- A number of armature conductors are connected in a specific manner to give armature winding.
- According to the way of connecting the conductors, the armature winding has two types,
  - Lap winding
  - Wave winding
- In lap type, the connections overlap each other as the winding proceeds.
- Due to this, the number of parallel path in which the conductors are divided is P, where P= number of poles in the machines
  - **A=P = Number of parallel paths**
- Large number of parallel path indicate high current capacity of machine hence lap winding is preferred for high current rating generators.



Lap Winding



Wave Winding

- In wave winding, the winding travels ahead avoiding the overlap in a progressive fashion.
- Due to this, the armature conductors always get divided into two parallel paths, irrespective of number of poles.
    - **A=2= number of parallel paths**

Comparison between lap and wave type armature winding

Sl. No.	Lap Winding	Wave Winding
1.	Number of parallel paths (A)= Poles (P)	Number of parallel paths (A) = 2
2.	Number of brush set required is equal to number of poles	Number of brush sets required is always equal to two
3.	Preferable for high current, low voltage capacity generators	Preferable for high voltage, low current capacity generators
4.	Normally used for generators of capacity more than 500A	Normally used for generators of capacity less than 500A

4.5.3 E.M.F Equation of D.C Generator

Let P = Number of poles of the generator

Φ = Useful flux per pole in Webers

N = Speed of the armature in r.p.m

Z = Total number of armature conductors

A = Number of parallel paths.

So A=P for lap type of winding and A=2 for wave type of winding

E.M.F gets induced in the conductor according to Faraday`s law of electromagnetic induction.

Hence the average value of e.m.f induced in each armature conductor = rate of cutting of flux = $\frac{d\varphi}{dt}$

Consider one revolution of conductor.

In one revolution, conductor will cut total flux produced by all the poles i.e. φP = dφ.

While time required to complete one revolution is 60/N sec = dt

$$\therefore \text{ EMF induced in one conductor} = \frac{\varphi P}{\frac{60}{N}} = \frac{\varphi P N}{60}$$

Now the conductors in one parallel path are always in series. There are total Z conductors with A parallel path, hence Z/A number of conductors are always in series and e.m.f remains same across all the parallel paths.

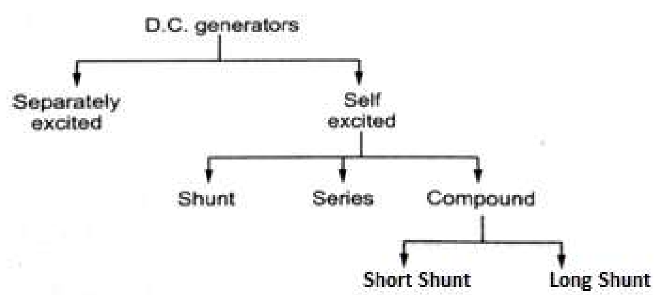
$$\therefore \text{ Total e.m.f can be expressed as, } E = \frac{\varphi PN}{60} \times \frac{Z}{A}$$

This is the e.m.f equation of a D.C. generator

$$\therefore \ \mathbf{E} = \frac{\varphi PNZ}{60} \quad \text{with A= P for lap winding and A=2 for wave winding}$$

4.5.4 Types of D.C. Generators

- The field winding is also called exciting winding and current carried by the field winding is called an exciting current.
- Thus supplying current to the field winding is called excitation and the way of supplying the exciting current is called method of excitation.
- Depending on the method of excitation used, the D.C. Generators are classified as,



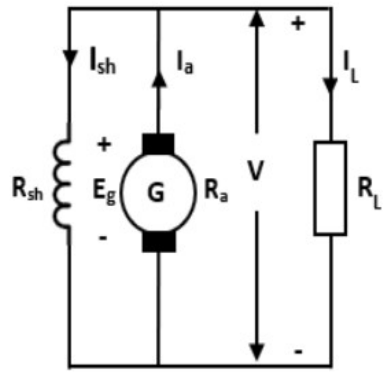
4.5.4.1 Self Excited Generator

- When the field winding is supplied from the armature of the generator itself then it is said to be self excited generator.
- Based on how field winding is connected to the armature to derive its excitation, this type is further divided into the following three types:

i) Shunt generator, ii) Series generator and iii) Compound generator

A. Shunt generator

- When the field winding is connected in parallel with the armature and the combination across the load then the generator is called shunt generator.



- From the fig

$$I_a = I_L + I_{sh}$$

- Now the voltage across the load is V, which is same across field winding as both are in parallel with each other.

$$I_{sh} = \frac{V}{R_{sh}}$$

- Hence the load voltage is given by,

$$\mathbf{V} = E_g - I_a R_a - BCD$$

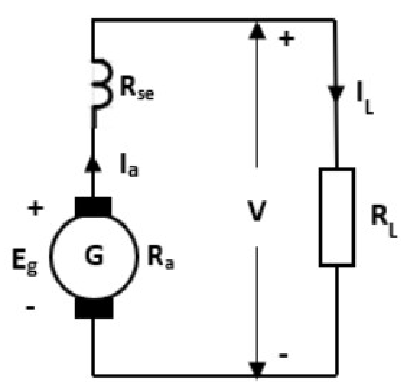
- Armature reaction drop is practically neglected.
- The power developed by armature is given by the product of induced e.m.f $\mathbf{E_g}$ and armature current I_a

Power developed in armature = $\mathbf{E_g} I_a$ Watts

- While the power available to the load is VI_L Watts.

B. Series Generator

- When the field winding is connected in series with the armature winding while supplying the load then the generator is called series generator.



- As all armature, field and load are in series they carry the same current

$$\therefore I_a = I_{se} = I_L$$

- Now in addition to drop $I_a R_a$, induced e.m.f has to supply voltage drop across series field winding too. This is $I_a R_{se}$ so voltage equation can be written as

$$\mathbf{V} = E_g - I_a(R_a + R_{se)} - BCD$$

$$\mathbf{V} = E_g - I_a(R_a + R_{se)} - BCD$$

4.5.5 Application of various type of D.C Generator

A. Separately Excited Generator

- As a separate supply is required to excite field, the use is restricted to some special applications like electro-plating, electro-refining of materials.

B. Shunt Generators

- Commonly used in battery charging and ordinary lighting purpose.

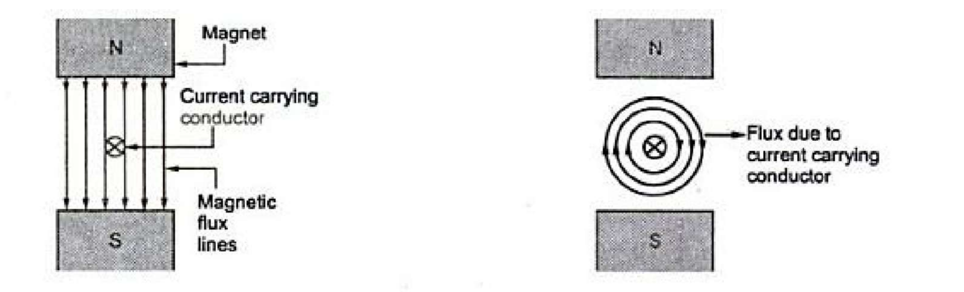
C. Series Generator

- Commonly used as boosters on d.c. feeders, as a constant current generator for welding

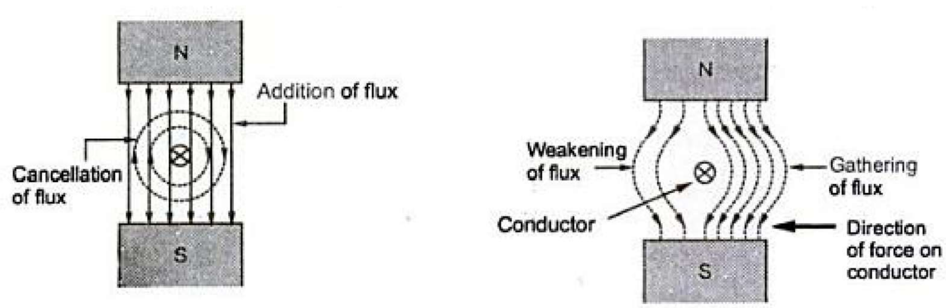
4.6 D.C. machine as a Motor

4.6.1 Introduction

- The principle of operation of a D.C. motor can be stated in a single statement as 'when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force'.
- In a practical d.c.motor, field winding produces a required magnetic field while armature conductors play a role of a current carrying conductor and hence armature conductors experience a force.
- Consider a single conductor placed in a magnetic field as shown in fig.
- Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it carries a current away from an observer as shown in fig.
- Any current carrying conductor produces its own magnetic field around it; hence this conductor also produces its own flux. The direction of this flux can be determined by right hand thumb rule. For direction of current considered the direction of flux around a clockwise.
- Now there are two fluxes present,
    - The flux produced by permanent magnet called main flux.
    - The flux produced by the current carrying conductor.



(a) Flux due to magnet (b) Flux due to current carrying conductor



(a) Presence of the two fluxes (b) Resultant flux pattern

- From this it is clear that on one side of the conductor, both the fluxes are in the same direction. In this case on the right hand side of the conductor there is gathering of the flux lines as two fluxes help each other.
- As against this on the left hand side of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this density of flux in this area gets weekend. So on the right, there exists high flux density area while on the left there exist low density area.
- This flux distribution around the conductor acts like a stretched rubber band under tension. This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area.
- In a practical d.c motor, the permanent magnet is replaced by a field winding which produces the required flux called main flux and all armature conductors, mounted on the periphery of the armature drum, gets subjected to the mechanical force.
- Due to this, overall armature experiences a twisting force called torque and armature of the motor starts rotating.
- The magnitude of force experienced by conductor in a motor is given by

$$F = B\,l\,I \text{ Newtons}$$

B= flux density due to the flux produced by field winding

l= active length of the conductor

I= magnitude of current passing through the conductor

- The direction of such force i.e. direction of rotation of motor can be determined by Flemings left hand rule.

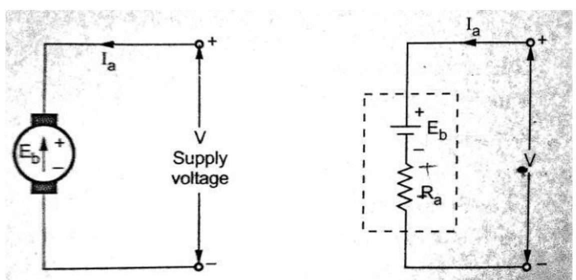
4.6.2 Back E.M.F in a D.C Motor

- It is seen in the generating action, that when a conductor cuts the lines of flux, e.m.f gets induced in the conductor. In a d.c motor, after a motoring action, armature starts rotating and the armature conductors cut the main flux. So there is generating action existing in a motor after motoring action.
- There is an induced e.m.f in the rotating armature conductors according to Faraday`s law of electromagnetic induction. This induced e.m.f in the armature always acts in the opposite direction to the supply voltage. This is according to Lenz`s law which states that the direction of the induced e.m.f is always so as to oppose the cause producing it.

- In a D.C. motor, electrical input i.e. supply voltage is the main cause for the armature current and motoring action and hence this induced e.m.f opposes the supply voltage. This e.m.f tries to set up a current through the armature which is in the opposite direction to that, which supply is forcing through the conductor.
- As this e.m.f always opposes the supply voltage, it is called back e.m.f and denoted as E_b. Though it is denoted as E_b, basically it gets generated by the generating action. So its magnitude can be determined by the e.m.f equation

$$E_b = \frac{\varphi PNZ}{60} \text{ volts}$$

- This e.m.f is shown schematically in the fig (a). So if V is supply voltage in volts and R_a is the value of the armature resistance, the equivalent electric circuit can be shown as in fig (b)



(a) Back e.m.f. in a d.c. motor (b) Equivalent circuit

4.6.3 Voltage Equation of D.C. Motor

- From the equivalent circuit, the voltage equation for a D.C Motor can be obtained as

$$V = E_b + I_a R_a + BCD$$

- The Brush Contact Drop(BCD) is practically neglected.
- Hence the armature current I_a can be expressed as

$$I_a = \frac{V - E_b}{R_a}$$

4.6.4 Significance of Back E.M.F

- Due to the presence of back e.m.f the d.c motor becomes a regulating machine. i.e. motor adjusts itself to draw the armature current just enough to satisfy the load demand.
- The basic principle of this fact is that the back e.m.f is proportional to speed, E_b **α N.**

- When the load is suddenly put on the motor, motor tries to slow down. So speed of the motor reduces due to which back e.m.f also decreases. So the net voltage across the armature, $(V - E_b)$ increases and motor draws more armature current.
- Due to the increased armature current, force experienced by the conductor and hence the torque on the armature increases. The increase in the torque is just sufficient to satisfy increased load demand.
- When the load on the motor is decreased, the speed of the motor tries to increase. Hence back e.m.f increases. This causes $(V - E_b)$ to reduce which eventually reduces the current drawn by the
- armature. The motor speeds stops increasing when the armature current is just enough to produce the less torque required by the new load.
- So back e.m.f regulates the flow of armature current and it automatically alters the armature current to meet the load requirement.

4.6.5 Power equation of a D.C Motor

- The voltage equation of a D.C. Motor is given by

$$V = E_b + I_a R_a$$

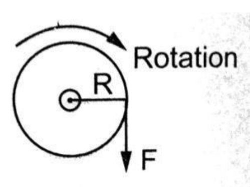
- Multiplying both sides of the above equation by I_a we get

$$V I_a = E_b I_a + I_a^2 R_a$$

- This equation is called power equation of d.c motor

4.6.6 Torque Equation of a D.C Motor

- The turning or twisting force about an axis is called torque.
- Consider a wheel of radius R meters acted upon by a circumferential force **F** Newton



- The wheel is rotating at a speed of **N** rpm then its angular speed is

$$\boldsymbol{\omega} = \frac{2\,\pi\,N}{60} \text{ rad/sec}$$

- So work done in one revolution is **W**= **F** x **distance travelled in one revolution** = **F** x **2 π R**

$$\textbf{P} = \textbf{Power developed} = \frac{work\ done}{time\ taken\ for\ one\ revolution} = \frac{F\ x\ 2\pi\ R}{60/N} = (\textbf{F} \text{ x } \textbf{R}) \text{ x } \frac{2\pi}{60}$$

$$\therefore \mathbf{P = T \times \omega} \text{ watts}$$

- Let T_a be the gross torque developed by the armature of the motor. It is also called armature torque.
- The gross mechanical power developed in the armature is $E_b I_a$.
- Hence, **Power developed in armature** = **Armature torque** x **ω**

$$E_b I_a = T_a \times \frac{2\pi N}{60}$$

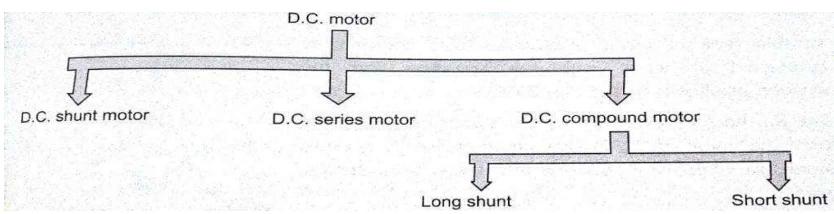
But $E_b = \frac{\varphi PNZ}{60A}$

$$\therefore \frac{\varphi PNZ}{60} \times I_a = T_a \times \frac{2\pi}{60}$$

$$\therefore T_a = \frac{1}{2\pi} \varphi I_a x \frac{PZ}{A}$$

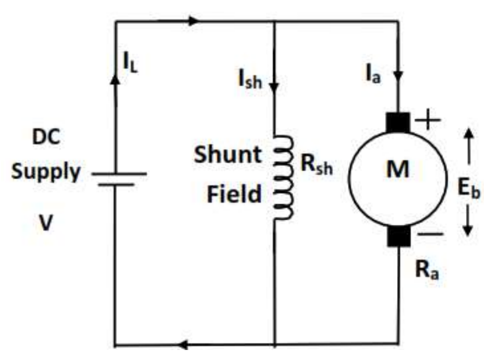
$$= \mathbf{0.159}\ \varphi I_a \cdot \frac{PZ}{A}$$

4.6.7 Types of D.C Motors



4.6.7.1 D.C Shunt Motor

- In this type, the field winding is connected across the armature winding and the combination is connected across the supply.



- Let R_{sh} be the resistance of shunt field winding and R_a be the resistance of armature winding.
- The value of R_a is very small while R_{sh} is quite large. Hence shunt field winding has more number of turns.

Voltage and current relationship

- The voltage across the armature and field winding is same equal to supply voltage **V**.
- The total current drawn from the supply is denoted as line current I_L.

$$I_L = I_a + I_{sh} \quad \text{and} \quad I_{sh} = \frac{V}{R_{sh}}$$

$$\mathbf{V} = E_b + I_a R_a + BCD$$

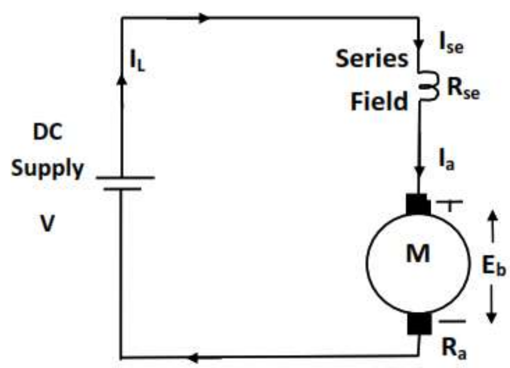
- Now flux produced by the field winding is proportional to the current passing through it. i.e. I_{sh}

$$\mathbf{\Phi \;\; \alpha} \;\; I_{sh}$$

- As long as supply voltage is constant, the flux produced is constant. Hence D.C. shunt motor is called constant flux motor.

4.6.7.2 D.C Series Motor

- In this type of motor, the series field winding is connected in series with the armature and the supply.
- Let R_{se} be the resistance of the series field winding and the value of R_{se} is very small.



Voltage and Current Relationship

- Let I_L be the total current drawn from the supply.

So, $I_L = I_{se} = I_a$

and $\mathbf{V} = E_b + I_a R_a + I_{se} R_{se} + BCD$

$$V = E_b + I_a(R_a + R_{se}) + BCD$$

- Supply voltage has to overcome the drop across series field winding in addition to E_b and drop across armature winding.
- In series motor, entire armature current is passing through the series field winding. So flux produced is proportional to armature current.

Φ α I_{se} **α** I_a for series motor.

4.6.8 CHARACTERISTICS OF DC MOTORS:

The characteristics of D.C motors are studied keeping the applied voltage constant. There are three important characteristics.

1. Armature torque vs. Armature current : T_a vs I_a (Electrical characteristics)
2. Speed vs. armature current characteristic: N vs. I_a
3. Speed vs. Torque: N vs. T_a (Mechanical characteristics)

4.6.8.1 CHARACTERISTICS OF D.C SHUNT MOTORS

1. **Armature torque vs. Armature current**

- From torque Equation

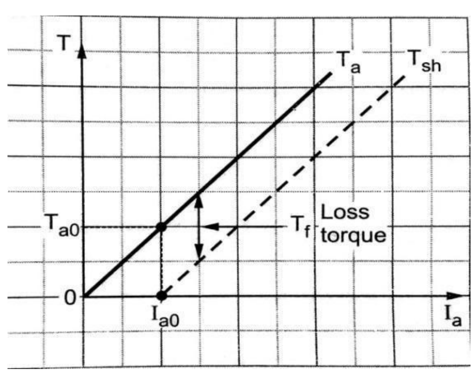
T_a α φ I_a

- Now **φ** is the flux produced by the field winding and is proportional to the current passing through the field winding

Φ α I_{sh}

- For a constant values of R_{sh} and supply voltage **V,** I_{sh} is also constant and hence flux is also constant.

∴ T_a **α** I_a



T_a vs I_a

- The equation represents a straight line passing through the origin.
- Torque increases linearly with armature current. So as load increases, armature current increases, increasing the torque developed linearly.

2. Speed vs. Armature current

- We have the back emf, $E_b = \frac{\varphi PNZ}{60A}$,

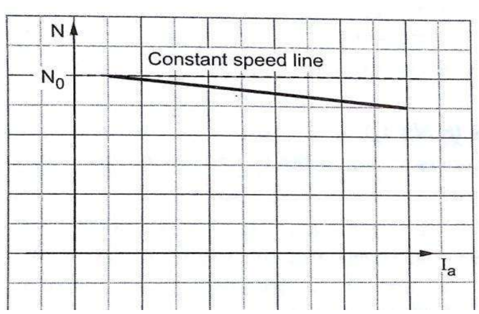
hence we can write the E_b **α φN** **i.e** **N α** $\frac{E_b}{\varphi}$

$\therefore E_b$ = **V-** $I_a R_a$ neglecting brush contact drop

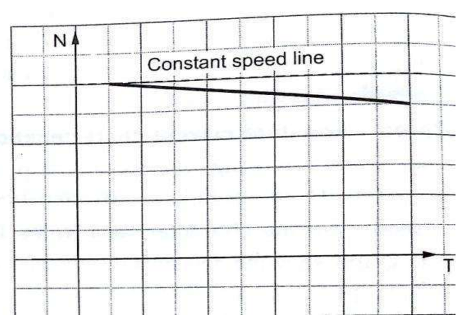
For shunt motor as flux φ is constant

N α V- $I_a R_a$

- So as load increases, the armature current increases and hence drop $I_a R_a$ also increases.
- But as R_a is very small, for change in I_a from no load to full load, drop $I_a R_a$ is very small and hence drop in speed is also not significant from no load to full load.



N vs I_a



N vs T_a

3. Speed vs. Armature torque

- These characteristic can be derived from the above two characteristic.
- This graph is similar to speed- armature current characteristic as torque is proportional to armature current.

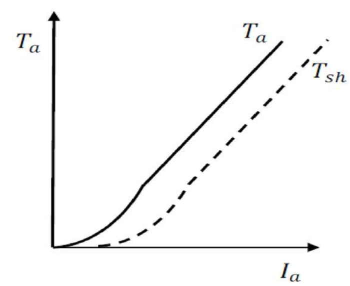
4.6.8.2 CHARACTERISTIC OF SERIES MOTOR

1. Armature torque vs. Armature current

- From torque Equation we have, **T_a α φ** I_a

Now φ is the flux produced by the field winding and is proportional to the current passing through the field winding. i.e, $\Phi \propto I_{se}$

- For the series motor the series field winding is carrying the entire armature current, hence $T_a \propto I_a^2$
- Thus torque in case of series motor is proportional to the square of the armature current. This relation is parabolic in nature.



T_a vs I_a

- As load increases armature current increases and torque produced increases proportional to the square of the armature current up to certain limit.
- As the entire armature current flows through series field, there is a property of electromagnet called saturation, may occur.
- Saturation means though the current through the winding increases, the flux produced remains constant. Hence after saturation the characteristic takes the shape of straight line as flux becomes constant.

2. Speed vs. Armature current

- We have the back emf, $E_b = \frac{\varphi PNZ}{60A}$,

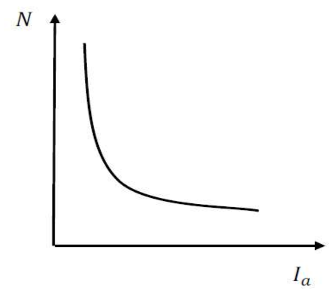
hence we can write the $E_b \propto \varphi N$ **i.e** $N \propto \frac{E_b}{\varphi}$

- For series motor, flux φ is proportional to I_a

Hence, $N \propto \frac{V - I_aR_a - I_aR_{se}}{I_a}$

- The values of R_a and R_{se} are so small that the effect of change in I_a on speed overrides the effect of change in $V - I_aR_a - I_aR_{se}$ on the speed.
- Hence in speed equation $E_b \approx \mathbf{V}$ and can be assumed constant.

- So speed equation reduces to **N α** $\frac{1}{I_a}$
- So speed armature current characteristic is rectangular hyperbola as shown in fig.



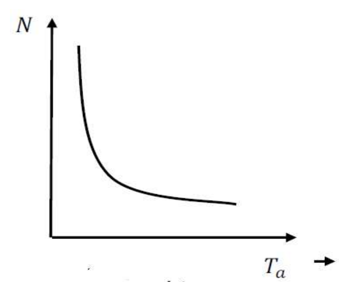
N vs I_a

3. Speed vs. Armature torque

- In case of series motor

$$T_a \; \alpha \; I_a^2 \quad \text{and} \quad N \; \alpha \; \frac{1}{I_a}$$

- Hence we can write, **N α** $\frac{1}{\sqrt{T}}$



N vs T_a

- When load increases, torque increases and hence the speed decreases.
- On no load torque is very less and hence speed increases to dangerously high value.
- Hence a D C series motor should never be started without load.

4.6.10 APPLICATIONS OF D C MOTOR:

a) D C Shunt Motor:

D C shunt motor has a medium starting torque and its speed remains almost constant from no load to full load. Hence it is used where constant speed is required with medium starting torque, such as lathes, centrifugal pumps, fans, reciprocating pumps, drilling machines, spinning and weaving machines etc.,

b) D C Series Motor:

D C series motor has a very high starting torque and its speed varies widely from no load to full load. Hence it is used where very high starting torque and variable speed is required, such as for electric traction work, electric locomotives, trolleys, cranes, hoists, conveyors, air compressors, vacuum cleaners, hair driers, sewing machines etc.,

c) D C Compound Motor:

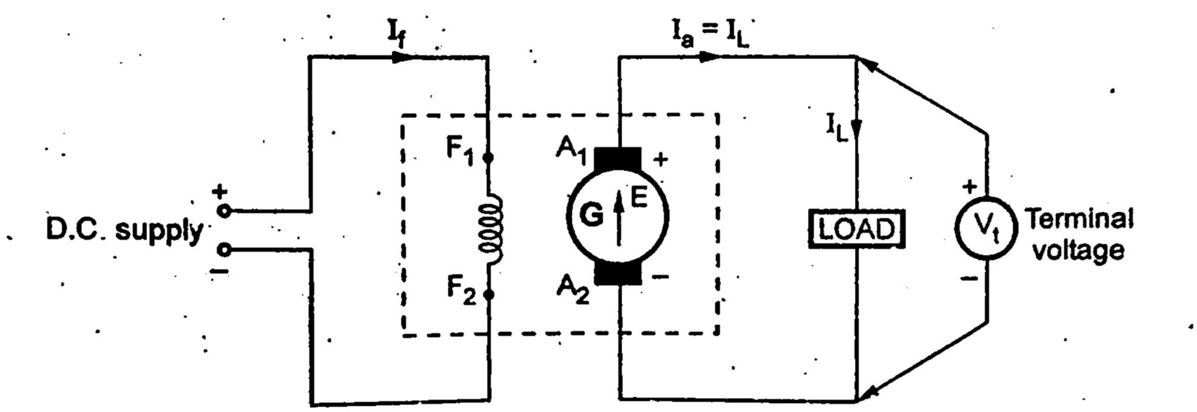
D C compound motors are used where sudden loads are applied or removed such as for shears, punches, coal cutting machines, elevators, conveyors, heavy planers, rolling mills, ice machines, printing presses, air compressors etc.,

SUMMARY OF CIRUITS AND EQUATIONS OF DC GENERATORS AND DC MOTORS

EMF INDUCED IN A DC GENERATOR =E= Eg= ϕZNP/60A

Φ=Flux per pole (Wb)
Z=No. of armature conductors
N=speed of rotation of armature conductors (rpm)
P=No. of poles
A=number of parallel paths
A=P for lap wound
A=2 for wave wound

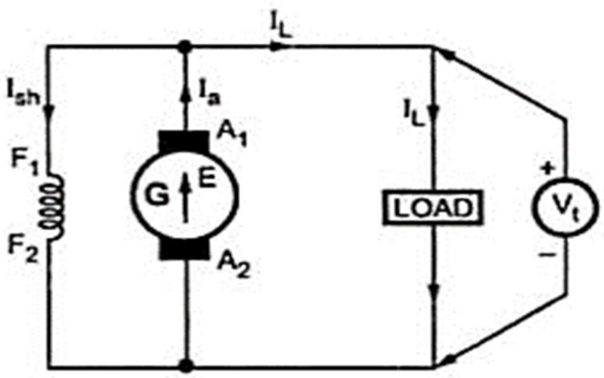
SEPERATELY EXCITED DC GENERATORS



E = **Vt** +**IaRa** + **Vbrush** + armature reaction drop

Where E = ϕPNZ/(60A) = generated e.m.f
Vt=Terminal voltage
Ia=Armature current
Ra=Armature resistance
$I_a = I_L$

DC SHUNT GENERATOR



E = ϕPNZ/(60A)

Ia= IL+Ish

Ish= Vt/Rsh

E = **Vt +IaRa + Vbrush**

Power developed in armature = (E * Ia) Watts

Power available to the load is (Vt * IL) watts

Where

Ia=Armature current

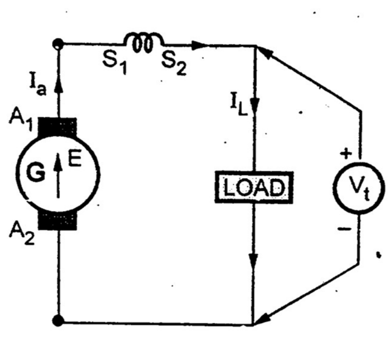
Ish=Current through shunt field winding

IL=Load current

Ra=Armature winding resistance

Rsh=Shunt field winding resistance

DC SERIES GENERATOR



E = ϕPNZ/(60A)

Ia= Ise= IL

E = **Vt +IaRa** + IaRse + **Vbrush**

Power developed in armature = (**E** * **Ia**) **Watts**

Power available to the load is (**Vt** * **IL**) **watts**

Where Ia=Armature current

Ise=Current through series field winding

IL=Load current

Ra=Armature winding resistance

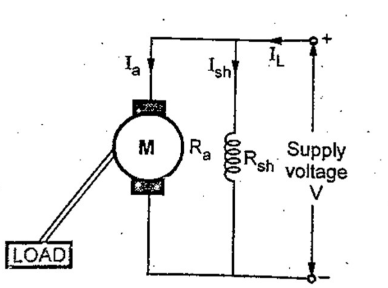
Rse=Series field winding resistance

DC MOTORS

Back emf= Eb = ϕPNZ/(60A)

Mechanical power in the armature = Eb * Ia watts

DC SHUNT MOTOR

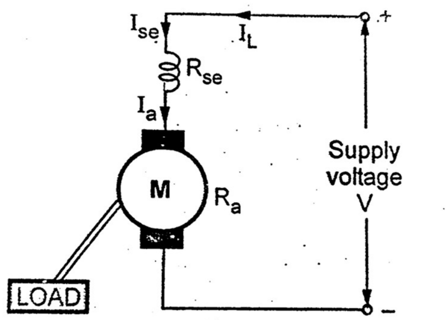


$$I_L = I_a + I_{sh} \quad \text{and} \quad I_{sh} = \frac{V}{R_{sh}}$$

$$\mathbf{V} = E_b + I_a R_a + BCD$$

OR **Eb = V- IaRa – BCD**

DC SERIES MOTOR



$$I_L = I_{se} = I_a$$

$$V = E_b + I_aR_a + I_{se}R_{se} + BCD$$

$$V = E_b + I_a(R_a + R_{se}) + BCD$$

OR **Eb = V – Ia(Ra + Rse) – BCD**

Torque Equation of DC motor

Ta=Armature torque = Gross torque **Tsh=shaft torque=useful torque**

$$T_a = \frac{1}{2\pi}\varphi I_a \frac{PZ}{A}$$

$$T_a = 0.159\varphi I_a \frac{PZ}{A} \quad Nm$$

$$T_a = \frac{E_b\, I_a}{2\pi\, N/60}$$

$$T_{sh} = \frac{Outputs\ in\ watts}{2\pi N/60}\ N - m\ in\ r.p.m$$

An 8-pole d.c. generator has 500 armature conductors, and a useful flux of 0.05 Wb per pole. What will be the e.m.f. generated if it is lap-connected and runs at 1200 rpm ? What must be the speed at which it is to be driven produce the same e.m.f. if it is wave-wound?

Solution. With lap-winding, P = A = 8

E = φ Z(N/60) (P/A) = 0.05 × 500 × 20 × 1= 500 volts for lap-winding

If it is wave-wound, P = 8, A = 2, P/A = 4

and E = 0.05 × 500 × (N/60) × 4

For E = 500 volts, N = 300 rpm

Hence, with wave-winding, it must be driven at 300 rpm to generate 500 volts.

The armature of a four-pole d.c. shunt generator is lap-wound and generates 216 V when running at 600 r.p.m. Armature has 144 slots, with 6 conductors per slot. If this armature is rewound, wave-connected, find the e.m.f. generated with the same flux per pole but running at 500 r.p.m.

Solution. Total number of armature conductors = Z = 144 × 6 = 864

For a Lap winding, number of parallel paths in armature = number of poles

In the e.m.f. equation, E = (φ ZN/60) (P/A)

Since E = φ ZNP/60A

216 = φ × 864 × 600/60 = 8640φ

Hence φ = 25 milli-webers

If the armature is rewound with wave-connection, number of parallel paths = 2.

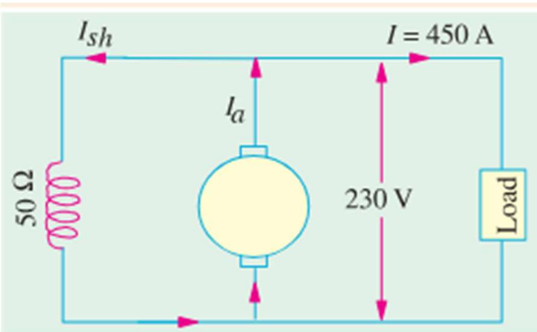
Hence, at 500 r.p.m., with 25 mWb as the flux per pole.

the armature emf = $(25 \times 10^{-3} \times 864 \times 500/60) \times 4/2$

= 25 × 864 × 0.50 × 2/60

= 360 volts

A shunt generator delivers 450 A at 230 V and the resistance of the shunt field and armature are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f. and the power delivered by the armature if there is a brush drop of 1V/brush



Solution

Current through shunt field winding is = Ish = Vsh/Rsh=230/50 = 4.6 A

Load current I = 450 A

∴ Armature current Ia = I + Ish = = 450 + 4.6 = 454.6 A

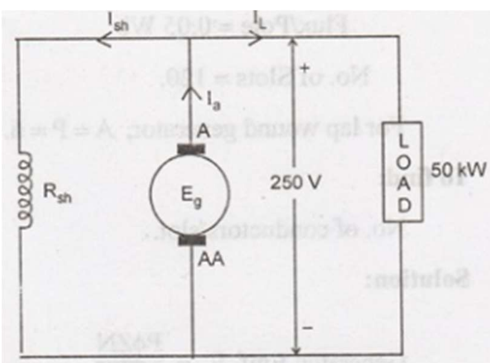
Armature voltage drop =I a Ra = 454.6 × 0.03 = 13.6 V

Now Eg = terminal voltage + armature drop = V + IaRa + Brush Drop [B.D= 2 x 1=2v]

∴ e.m.f. generated in the armature Eg = 230 + 13.6 + 2 = 245.6 V

Armature power = Eg*Ia=245.6 * 454.6 = 111.6 KW

A 50 kW, 250 V shunt generator operates on full load at 1500 rpm. The armature has 6 poles and is lap wound with 200 turns. Find the induced emf and the flux / pole at full load. Given that the armature and field resistances are 0.01 and 125 Ω respectively. Neglect armature reaction. Also find the power delivered by the armature



$$I_L = \frac{50\times10^3}{250} = 200\text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2\text{ A.}$$

For a shunt generator,

$$I_a = I_L + I_{sh}$$

$$= 202\text{ A.}$$

Induced emf $E_g = V + I_a R_a$

$$= 250 + 202 \times 0.01$$

$$= 252.02\text{ V}$$

$$E_g = 252.02\text{ Volts.}$$

$Z = 200 \times 2$ (1 turn = 2 conductors)

$N = 1500$ rpm ; $P = A = 6$ (lap wound)

$$\phi = \frac{E_g\,60\,A}{PZN} = \frac{252.02\times60\times6}{400\times1500\times6}$$

$\phi = 0.025205$ Wb (or)

$\boxed{\phi = 25.205\text{ mWb}}$

Pa=Eg*Ia=252.02 * 202 = 50.91 KW

Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

Solution. Developed torque or gross torque is the same thing as armature torque.

∴ Ta = 0.159 Φ ZA (P/A) = $0.159 \times 25 \times 10^{-3} \times 800 \times 45\ (4/2) = 286.2$ N-m

Eb = V − Ia Ra = $220 - 45 \times 0.6 = 193$ V

Now, Eb = Φ ZN (P/60A) or $193 = 25 \times 10^{-3} \times 800 \times N\ \pi \times (4/2)$

∴ N = 4.825 r.p.s.=290 rpm

Also,

2πN *Tsh = output or 2π × 4.825 Tsh = 8200 ∴ Tsh = 270.5 N-m

A 250-V, 4-pole, wave-wound d.c. series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 ohm. The motor takes a current of 40 A. Estimate its speed and gross torque developed if it has a flux per pole of 25 mWb.

Solution.

Eb = ΦZN (P/60A)

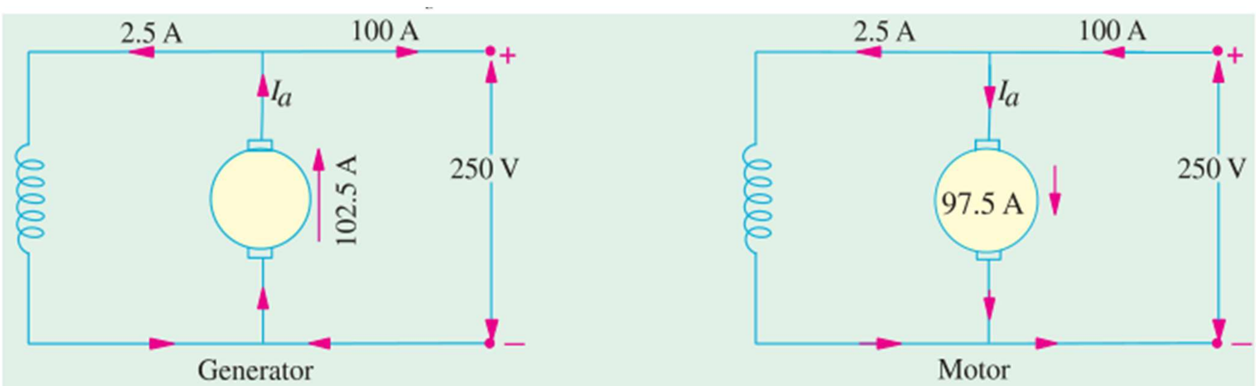
Now, Eb = V − IaRa = 50 − 40 × 0.75 = 220 V

∴ $220 = 25 \times 10^{-3} \times 782 \times N \times 0.75$ hence N=338 rpm

∴ Ta = 0.159 Φ ZIa (P/A) = $0.159 \times 25 \times 10^{-3} \times 782 \times 40 \times (4/2) = 249$ N-m

---

A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of 0.06 Ω and 100 Ω respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.



Solution. As Generator

Output current = 25,000/250 = 100 A ; Ish = 250/100 = 2.5 A ; Ia = 102.5 A

Generated e.m.f. = 250 + Ia Ra = 250 + 102.5 × 0.06 = 256.15 V

Power developed in armature = EbIa = (256.15 *102.5)= 26.25 kW

As Motor

Motor input current = 100 A ; Ish = 2.5 A, Ia = 97.5 A

Eb = 250 − (97.5 × 0.06) = 250 − 5.85 = 244.15 V

Power developed in armature = Eb Ia = 244.15 × 97.5 = 23.8 kW

An 8-pole d.c. shunt generator with 778 wave-connected armature conductors and running at 500 r.p.m. supplies a load of 12.5 Ω resistance at terminal voltage of 50 V. The armature resistance is 0.24 Ω and the field resistance is 250Ω. Find the armature current, the induced e.m.f. and the flux per pole.

Solution. Load current = V/R = 250/12.5 = 20 A

Shunt current = 250/250 = 1 A

Armature current = 20 + 1 = 21 A

Induced e.m.f. = 250 + (21 × 0.24) = 255.04 V=Eg

Now, E g = Φ ZNP/60A

∴ Φ = 9.83 mWb

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