

MODULE – 1: Lasers & Fiber Optics

Lasers - Expression for Energy Density using Einstein Coefficients, Condition for Laser Action, Requisites for Laser System, Construction and Working of Hetero-Junction Semiconductor Diode Laser, Construction and Working of LIDAR.

Optical Fibers: Structure, Principle and Light guiding mechanism, Acceptance Angle and Numerical Aperture, V-number and Modes of Propagation, Attenuation and their Types, Mach-Zehnder Interferometer and its Applications.

LASER

LASER is the acronym for Light Amplification by Stimulated Emission of Radiation.

The principle of LASER is Stimulated Emission.

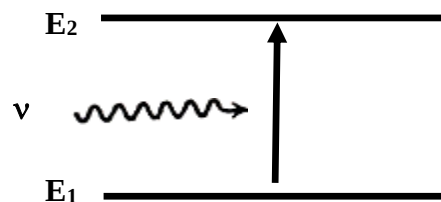
Laser is a unique beam of light with basic properties of

- ❖ LASER light is nearly a monochromatic beam
- ❖ It is a unidirectional beam
- ❖ It is a coherent beam where all waves are exactly in phase with one another
- ❖ It consists of high directionality and hardly diverges.
- ❖ Laser is extremely intense; hence, with laser, we can achieve very high energy density.

Matter-Radiation Interaction:

When radiation interacts with matter, it leads to the transition of a quantum system such as an atom or molecule from one energy state to another.

Consider two energy states E_1 and E_2 , ($E_2 > E_1$) of an atomic system. An electron at the energy state E_1 is excited to E_2 , when it absorbs a light photon of energy $\Delta E = (E_2 - E_1)$. If an electron makes a transition from the higher energy state E_2 , a light photon of energy $\Delta E = E_2 - E_1$ is emitted. In both cases, the frequency of the photon involved is $\nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$



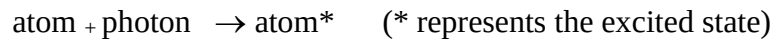
There are three possible ways in which this interaction of radiation and matter can take place.

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1. Induced (stimulated) absorption

Induced absorption is a process in which an atom in a lower energy state rises to a higher energy state by absorbing a suitable photon.

Induced absorption can be represented as



2. Spontaneous emission:

Spontaneous emission is a process in which an atom in the higher energy state falls to a lower state by emitting a photon on its own.

This process can be represented as

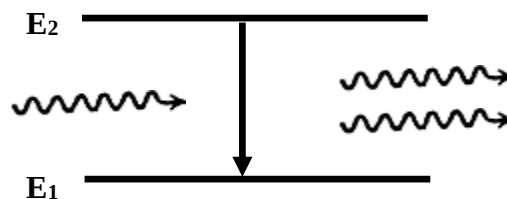


Note: 1) Life time of atoms in higher energy state is usually very small of the order of 10^{-8} sec.

2) If there is an assembly of atoms, the radiation emitted spontaneously by each atom has a random direction and random phase. Therefore, radiation emitted by spontaneous emission is incoherent. (Eg: glowing electric bulb)

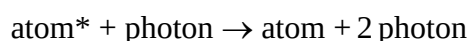
3. Stimulated emission:

Stimulated emission is a process of the emission of a photon by a system under the influence of an incident photon of suitable energy, due to which the system transits from a higher energy state to a lower energy state.



Consider an atom in the excited state with energy E_2 . Let a photon of energy $\Delta E = E_2 - E_1$ interacts with this atom. As a result, the atom emits a photon and transits to the lower energy state. The emitted photon will have the same phase, energy and direction of movement as that of the incident photon.” The electromagnetic waves associated with the two photons will have same phase and thus they are coherent. **This kind of emission is responsible for laser action.**

This process can be represented as



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Expression for Energy density in terms of Einstein's coefficients:

Consider two energy states E_1 and E_2 of a system of atoms ($E_1 < E_2$). Let N_1 and N_2 be the number of atoms in these energy states per unit volume of the system. Let $U_\nu d\nu$ be the energy of the incident radiation per unit volume of the system, where radiation lies in the frequency range ν to $\nu + d$, then ' U_ν ' is called the energy density of frequency " ν ".

$$\text{The rate of stimulated absorption} = B_{12}N_1U_\nu \rightarrow (1)$$

Where $B_{12} \rightarrow$ Einstein's Coefficient of induced absorption.

$$\text{The rate of spontaneous absorption} = A_{21}N_2 \text{ ----- } (2)$$

where $A_{21} \rightarrow$ Einstein's coefficient of spontaneous emission.

$$\text{The rate of stimulated absorption} = B_{21}N_2U_\nu \text{ ----- } (3)$$

where B_{21} is Einstein's coefficient of stimulated emission.

At thermal equilibrium,

the rate of absorption = rate of spontaneous emission + rate of stimulated emission.

$$\text{ie } B_{12}N_1U_\nu = A_{21}N_2 + B_{21}N_2U_\nu$$

$$\text{ie } U_\nu(B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

$$\therefore U_\nu = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$U_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}N_1}{B_{21}N_2} - 1} \right] \text{ ----- } (4)$$

According to Boltzmann's law we have

$$\frac{N_1}{N_2} = e^{h\nu/kT} \text{ ----- } (5)$$

Therefore equation (4) becomes

$$U_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \right] \text{ ----- } (6)$$

According to Planck's law of radiation

$$U_\nu = \frac{8\pi h\nu^3}{c^3} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right] \text{ ----- } (7)$$

Comparing equation (6) and (7) we have

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$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}, \text{ and } \frac{B_{12}}{B_{21}} = 1$$

$$\rightarrow B_{12} = B_{21} = B \quad \text{----- (8) and } A_{21} = A$$

The identity (8) implies that the probability of induced absorption is equal to the probability of stimulated emission.

Therefore, we can write the expression for energy density in terms of Einstein's A & B coefficients as

$$U_\nu = \frac{A}{B \left[e^{\frac{h\nu}{kT}} - 1 \right]}$$

Condition for Laser action:

1. Population inversion:

Population inversion is the state of an atomic system in which the number of atoms in the higher energy level is greater than the number of atoms in the lower energy state.

2. Metastable state:

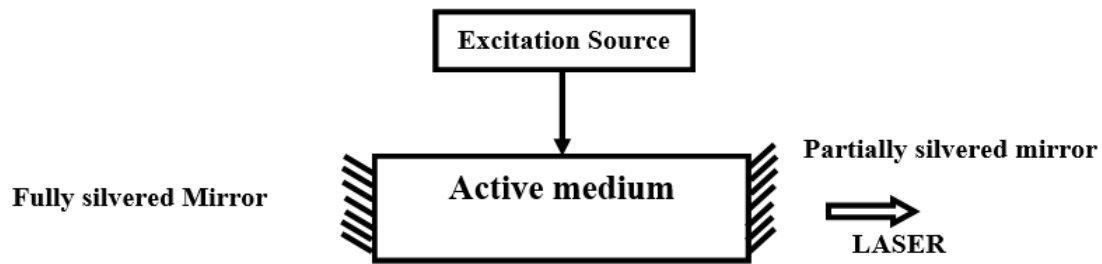
An excited energy state between the ground state and one of the ordinary excited states having a longer lifetime of about 10^{-3} to 10^{-2} sec is called the **metastable state**.

Requisites of a laser system

- 1) An **excitation source** provides energy in an appropriate form for pumping the atoms to higher energy levels. If the pumping is achieved by light energy input, then it is called optical pumping (Ruby laser). If the pumping is achieved by electrical energy input, then it is called electrical pumping (He-Ne laser).
- 2) **Active medium.** A medium in which light gets amplified is known as an active medium. The medium may be solid, liquid, or a gas. It contains atoms having at least one metastable state along with the ground state and other ordinary excited states.
- 3) **Laser cavity (Resonating cavity):**

A laser device consists of an active medium bound between two parallel mirrors, one is perfect reflector and other one is partially reflector. The mirrors reflect the photon to and fro through the active medium. This arrangement is known as a laser cavity.

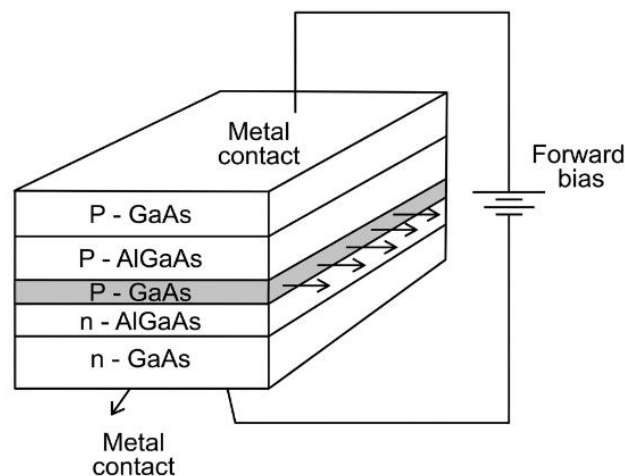
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Inside the cavity, two types of waves exist, one moving towards the right and the other to the left. These waves interfere constructively or destructively depending on the phase difference. To arrange for constructive interference, the distance 'L' between the two mirrors should be such that the cavity supports an integral number of half wavelengths, i.e. $L = m \frac{\lambda}{2}$, where m is an integer. This results in the amplification of stimulated emission of radiation, which is the **laser light**.

Hetero junction Semiconductor Diode Laser:

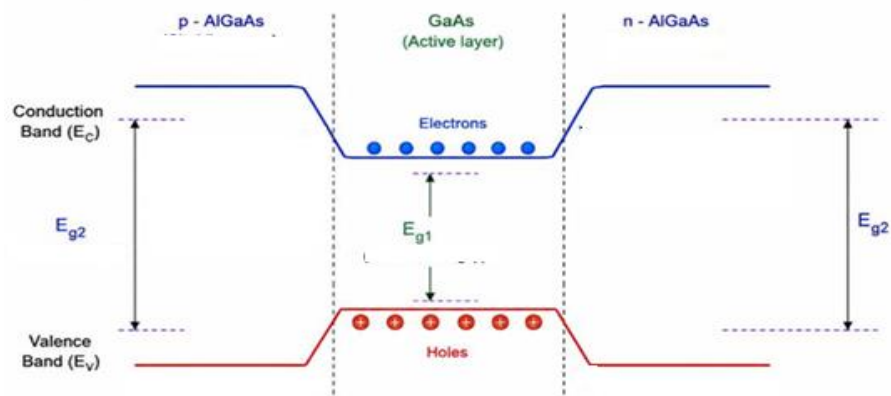
Construction:



The heterojunction laser diode consists of five layers, as shown in the figure. The active region is made of p-type GaAs with a narrow energy gap and a high refractive index. This layer is sandwiched between two layers of p-type GaAlAs and n-type GaAlAs, which have a wider energy gap and a lower refractive index. The opposite ends of the diode, perpendicular to the junction, are well polished to serve as parallel reflective mirrors, forming an optical cavity. The top p-type GaAs layer provides a low-resistance ohmic contact, and a metal contact is deposited on it. The bottom n-type GaAs layer acts as a substrate, and a metal contact is attached to it.

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Working:



When the junction is formed, the conduction band decreases in the active region while the valence band increases. The energy band diagram of a forward-biased laser diode is as shown above. When a suitable forward bias is applied, electrons and holes are injected into the active region from the n-type and p-type GaAlAs regions, respectively. This results in a large number of electrons in the conduction band and a large number of holes in the valence band, achieving population inversion. At this stage, a few electrons and holes recombine spontaneously in the active region. These photons will trigger stimulated recombination, releasing more in-phase photons. The photons emitted by stimulated emission reflect back and forth between the mirrors, increasing the intensity of the beam. When the intensity increases to the desired level, the laser beam emerges through the partially reflecting mirror. The wavelength of the emitted beam depends on the band gap energy (E_g) and is given by

$$\lambda = \frac{hc}{E_g}$$

Advantages:

1. It produces continuous wave output at room temperature.
2. The power output is very high.
3. The efficiency of the device is high.

Disadvantages:

1. It is difficult to grow different layers of the PN junction.
2. The cost of the device is high.
3. Wavelength and efficiency vary with the temperature.

Applications of Laser:

Light Detection and Ranging (LIDAR):

Principle:

LiDAR (Light Detection and Ranging) is based on the **Time-of-Flight (ToF)** principle, in which the distance to an object is determined by measuring the time taken by a laser pulse

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to travel between the target and transmitter or the receiver. It is an advanced optical sensing device that uses **laser pulses** to measure the distance, shape, and position of objects in the surrounding environment. The distance of the object is calculated using:

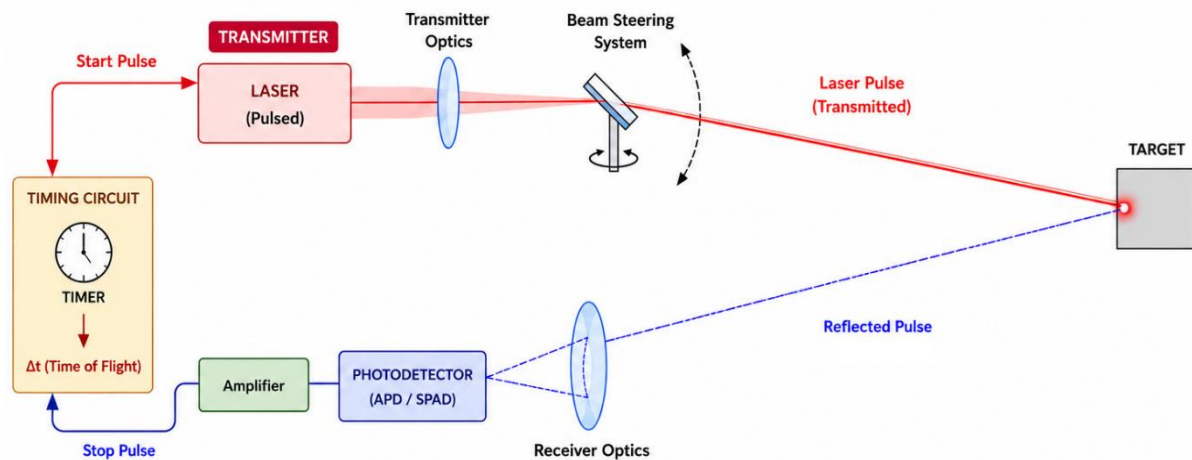
$$d = \frac{c \times t}{2} \quad (1)$$

where,

d = distance of the object, **c** = velocity of light, **t** = time taken by the laser pulse to travel from transmitter to target and back to the receiver.

Construction & Working:

The schematic diagram showing the main components of a LIDAR is given below.



A LASER pulse from a laser source is collimated by the transmitter optics, which is scanned across the target by the beam steering system. Beam steering is a mechanism that directs the laser beam in different directions to scan the environment. This laser pulse strikes the target and is reflected. The reflected laser pulse is collected by the receiver optics and focused onto the detector. The light beam from the receiver is detected by the highly sensitive photodetectors, which produce a weak electrical signal. The signal is then amplified and sent to the timing circuit, which measures the time of flight (**t**) of the laser pulse. Using equation (1), the distance “**d**” is calculated. By scanning the laser beam in multiple directions and repeating this process, the signal processing unit built into the LiDAR system generates a three-dimensional point cloud of the surrounding environment.

Applications of LIDAR:

LIDAR is used in

1. Self-driving and autonomous vehicles.
2. Measuring atmospheric pollution
3. Landing guidance systems
4. Earth and land surveys

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Solved Numerical Problems:

1. A medium in thermal equilibrium at 325 K has two energy levels with a wavelength separation of $1\mu\text{m}$. Find the ratio of population densities of the upper and lower levels.

Given: Thermal equilibrium, $T = 325\text{ K}$

Wavelength separation, $\lambda = 1 \times 10^{-6}\text{ m}$

To Find: $\frac{N_2}{N_1} = ?$

Solution: we know that $\frac{N_2}{N_1} = e^{-\frac{hc}{\lambda kT}}$

$$\frac{N_2}{N_1} = e^{-\frac{(6.623 \times 10^{-34}) \times (3 \times 10^8)}{1.38 \times 10^{-23} \times 325 \times 1 \times 10^{-6}}}$$

$$\frac{N_2}{N_1} = 5.68 \times 10^{-20}$$

$\therefore$ The ratio of population densities = 5.68×10^{-20}

2. The ratio of populations of two energy levels is 1.059×10^{-30} . Find the wavelength of the light emitted at 340 K.

Given: Thermal equilibrium, $N_2/N_1 = 1.059 \times 10^{-30}$

To find: Wavelength of light, $\lambda = ?$

Solution: We know that

$$\frac{N_2}{N_1} = e^{-\frac{h\nu}{kT}}$$

$$1.059 \times 10^{-30} = e^{-\frac{(6.623 \times 10^{-34}) \times (3 \times 10^8)}{1.38 \times 10^{-23} \times 340 \times \lambda}}$$

$$\lambda = 613.7\text{ nm}$$

$\therefore$ The wavelength of light emitted = 613.7 nm

3. Find the number of modes of standing waves and their frequency Separation in the resonant cavity of 1 m length of He-Ne laser operating at a wavelength of 632.8 nm.

Solution:

For constructive interference in a laser cavity, we have $L = m \frac{\lambda}{2}$,

$$\begin{aligned} \text{The number of modes supported by the cavity } m &= \frac{2L}{\lambda} \\ &= \frac{2 \times 1}{632.8 \times 10^{-9}} = 3.16 \times 10^6 \text{ modes} \end{aligned}$$

Frequency separation between the adjacent modes:

The frequency of oscillation is given by $\nu = \frac{mc}{2L}$

Let ν_m and ν_{m-1} be the frequency of the m^{th} and $(m-1)^{\text{th}}$ mode, then

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$$\nu_m - \nu_{m-1} = \frac{mc}{2L} - \frac{(m-1)c}{2L}$$

$$\nu_m - \nu_{m-1} = \frac{c}{2L} = \frac{3 \times 10^8}{2 \times 1}$$

$$\nu_m - \nu_{m-1} = 1.5 \times 10^8 \text{ Hz}$$

4. A Ruby laser emits light of wavelength 694.3 nm. The resonant cavity length is 1.5 m. Calculate: a) The frequency separation (mode spacing).

b) The number of longitudinal modes.

Solution:

a) The frequency separation

$$\text{we have } \nu = \frac{mc}{2L}$$

Let ν_m and ν_{m-1} be the frequency of the m^{th} and $(m-1)^{th}$ mode, then

$$\nu_m - \nu_{m-1} = \frac{mc}{2L} - \frac{(m-1)c}{2L}$$

$$\nu_m - \nu_{m-1} = \frac{c}{2L} = \frac{3 \times 10^8}{2 \times 1.5}$$

$$\nu_m - \nu_{m-1} = 1.0 \times 10^8 \text{ Hz} = 100 \text{ MHz}$$

b) The number of longitudinal modes

$$\text{we have } L = m \frac{\lambda}{2}$$

The number of longitudinal modes supported by the cavity $m = \frac{2L}{\lambda}$

$$= \frac{2 \times 1.5}{694.3 \times 10^{-9}} = 4.32 \times 10^6 \text{ modes}$$

5. A semiconductor has a peak emission radiation of wavelength 0f 1.24μm. What is the band gap energy value in eV?

Solution:

$$\text{We have; } \lambda = \frac{hc}{E_g}$$

$$E_g = \frac{hc}{\lambda}$$

$$E_g = \frac{(6.623 \times 10^{-34}) \times (3 \times 10^8)}{1.24 \times 10^{-6}}$$

$$E_g = 1.6 \times 10^{-19} \text{ J}$$

$$E_g = \frac{1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$E_g = 1 \text{ eV.}$$

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Sample Questions:

1. Derive an expression for the energy density of radiation in terms of Einstein's A & B coefficients. (L2).
2. Define stimulated absorption, spontaneous emission, and stimulated emission. Explain the conditions and requisites for laser action. (L2)
3. With neat and relevant diagrams, describe the construction and working of a heterojunction semiconductor laser diode. List its advantages and limitations. (L2)
4. Explain the construction and working of LIDAR. (L2). Mention its applications
5. Find the ratio of the population of two energy levels in a LASER if the transition between them produces light of wavelength $6493 \text{ \AA}$, assuming the ambient temperature is 27°C . (L3)

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OPTICAL FIBERS

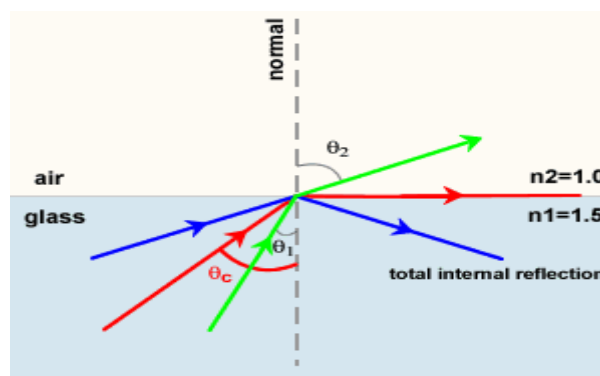
Introduction:

A conventional method of long-distance communication uses radio waves (10^6 Hz) and microwaves (10^{10} Hz) as carrier waves. A light beam acts as a carrier wave, which is capable of carrying far more information since optical frequencies are extremely large (10^{15} Hz).

Optical fibers are devices used in optical communication as waveguides. With the development of lasers and flexible fiber, optical fibers are being used extensively for various communication applications. The principle behind the transmission of light waves in an optical fiber is **total internal reflection**. Fiber optic communication has significant advantages over transmission by conventional coaxial metallic cables. The loss of signal strength is considerably less in optical fibers and hence permits transmission over long distances. Use of light waves in place of radio and microwave has improved the speed of communication.

Basic principle:

The basic principle of optical fiber is multiple **total internal reflections**. When a ray of light travels from a denser to a rarer medium, at an angle of incidence greater than the **critical angle** (θ_c) for the pair of media, the ray is reflected into the denser medium. This property is called **total internal reflection**. Light signals are transmitted through optical fibers by multiple total internal reflections.



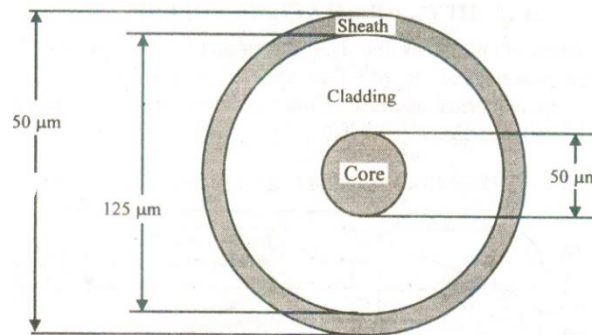
Structure of the optical fiber:

An optical fiber is a transparent fiber made of glass or plastic. It is designed to guide light waves along its length. An optical fiber works on the principle of **total internal reflection**. When light enters one end of the fiber, it undergoes successive total internal reflections from the sidewalls and travels down the length of the fiber along a zigzag path.

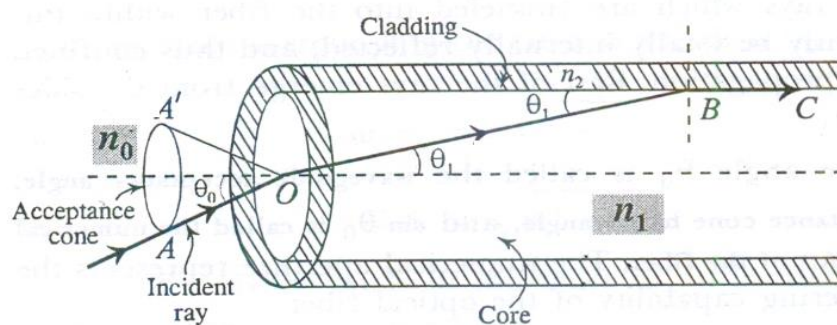
A practical optical fiber has, in general, three coaxial regions. The innermost region is the light-guiding region known as the **core**. It is surrounded by a coaxial region known as **cladding**. The outermost region is called the **sheath**. The refractive index (RI) of the core (denoted by n_1) is always greater than that of the cladding (denoted by n_2). The purpose of

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cladding is to confine the light to the core. The sheath protects the cladding and the core from contamination, abrasions, harmful influence of moisture. It also provides mechanical strength to the fiber.



Light guiding mechanism in optical fibers:



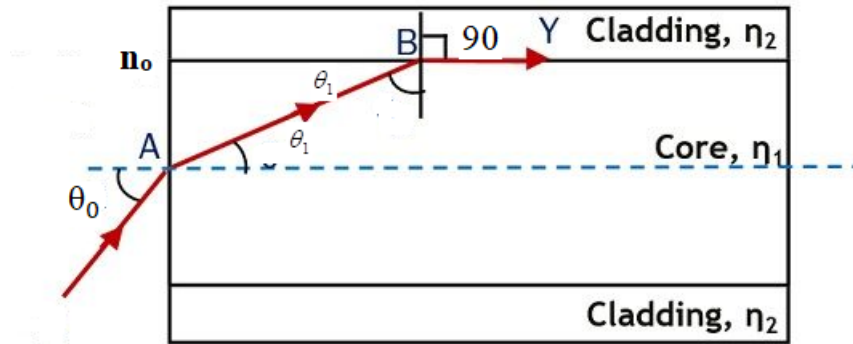
Consider an optical fiber into which light is launched at one end from a medium of RI n_0 . Let n_1 be the RI of the core and n_2 be that of the cladding. Let a ray of light be launched into the fiber at an angle θ_i with respect to the axis of the fiber. The light ray refracts at an angle and strikes the core – cladding interface at an angle of $90^\circ - \theta_1$. If $90^\circ - \theta_1$ is greater than the critical angle for the core – cladding interface, the ray undergoes total internal reflection at the interface. Let the angle θ_i is adjusted to a particular angle such that $90^\circ - \theta_1$ is equal to the critical angle for the core – cladding interface. Therefore, all light rays entering into the fiber at angles less than θ_0 will undergo total internal reflection when incident at the core-cladding interface.

The angle θ_0 is called the **acceptance angle**. *“It is the maximum angle that a light ray can have relative to the axis of the fiber and propagate down the fiber”*.

The sine of the acceptance angle θ_0 ($\sin \theta_0$) is called the **numerical aperture (NA)** of the fiber. It represents the light-gathering capacity of the optical fiber.

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Expression for Angle of Acceptance and Numerical Aperture:



Consider an optical fiber into which light is launched from a medium of RI n_0 . Let n_1 be the RI of the core and n_2 be that of the cladding.

Let a ray of light enter the fiber at an angle θ_0 known as the acceptance angle with respect to the axis of the fiber. The light ray refracts at an angle θ_1 and strikes the core – cladding interface at an angle of $(90 - \theta_1)$ which is equal to the critical angle for the core – cladding interface.

Applying Snell's law at 'A'

$$\frac{\sin \theta_0}{\sin \theta_1} = \frac{n_1}{n_0}$$

$$\sin \theta_0 = \frac{n_1}{n_0} (\sin \theta_1)$$

$$\sin \theta_0 = \frac{n_1}{n_0} (\sqrt{1 - \cos^2 \theta_1}) \text{ --- (1)}$$

Applying Snell's law at 'B'

$$n_1 \sin(90 - \theta_1) = n_2 \sin 90$$

$$\cos \theta_1 = \frac{n_2}{n_1} \text{ --- (2)}$$

Substituting (2) in (1) we get,

$$\sin \theta_0 = \frac{n_1}{n_0} \left(\sqrt{1 - \left(\frac{n_2}{n_1} \right)^2} \right)$$

$$\sin \theta_0 = \frac{n_1}{n_0} \left(\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} \right)$$

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

By definition, sine of the angle of acceptance is known as numerical aperture (N.A)

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$$N.A. = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If the medium surrounding the fiber is air, then $n_0 = 1$

$$N.A. = \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

If θ_i is the angle of incidence of an incident ray w.r.t. the axis of the fiber, then ray will be able to propagate,

if $\theta_i < \theta_0$

if $\sin \theta_i < \sin \theta_0$

or $\sin \theta_i < N.A.$

This is the **condition for propagation**.

Modes of propagation:

Light propagates as an electromagnetic wave through an optical fiber. All waves having ray directions above the critical angle will be trapped within the fiber due to total internal reflection. But all such waves do not propagate along the fiber. There is certain ray directions allowed for propagation. ***These allowed ray directions or possible number of paths of light in optical fibers are known as modes of propagation.*** The paths are zigzag paths except the one along the axial direction. The number of modes that a fiber will support depends on the diameter of the core and the wavelength of the wave being transmitted.

Normalized frequency (V – number):

It is the relation between fiber size, the refractive indices, and the wavelength of light propagating through the fiber. It is given by (when placed in air)

$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$$

Where, $d \rightarrow$ diameter of the core; $n_1 \rightarrow$ RI of the core; $n_2 \rightarrow$ RI of the cladding;
 $\lambda \rightarrow$ Wavelength of light.

Since $\sqrt{n_1^2 - n_2^2} = N.A.$ we can write

$$V = \frac{\pi d}{\lambda} NA$$

The number of modes supported by a fiber is given by

$$M_N = \frac{V^2}{2}$$

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The refractive index profile is a curve that represents the variation of refractive index with respect to the radial distance from the axis of the fiber.

Attenuation:

The loss of power suffered by the optical signal as it propagates through the fiber is called **attenuation**.

The attenuation or fiber loss is due to the following factors:

1. Absorption losses
2. Scattering losses
3. Radiation or bending losses

1. Absorption Losses:

The loss of signal strength occurs due to absorption of photons during its propagation. Photons are absorbed by

- a) Impurities in the silica glass of which the fiber is made.
- b) Intrinsic absorption by the glass material itself.

a. Absorption by impurities:

The impurities that are generally present in fiber glass are ions of OH, iron, chromium, cobalt, copper, etc. During signal propagation, when photons interact with these impurities, the electrons absorb the photons, resulting in loss of optical power.

b. Intrinsic Absorption

The fiber material itself has a tendency to absorb light energy; however small it may be. Hence, there will be a loss, which is known as intrinsic absorption.

2. Scattering Losses:

The optical power is lost due to the scattering of photons. This scattering is due to the non-uniformity in the density of the fiber material, which leads to the variation in the RI of the fiber. Structural inhomogeneities and defects created in the fiber can also cause scattering. The loss of light energy by scattering is found to be wavelength-dependent (Rayleigh Scattering). It decreases with an increase in the wavelength of light transmitted through the fiber.

3. Bending Losses (Radiation Losses):

Radiation losses occur due to the bending of the fiber. There are two types of bends

- a) Microscopic bends
- b) Macroscopic bends

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Microscopic bends are caused during manufacturing as well as due to the applied stress on the fiber. Macroscopic bending arises during the installation of the fiber. At the point of a bend, light will escape to the surrounding medium because the angle of incidence at that point becomes less than the critical angle.

To minimize these losses, the optical fiber has to be laid without sharp bends, and it should be freed from external stresses by providing mechanical strength through external coverage.

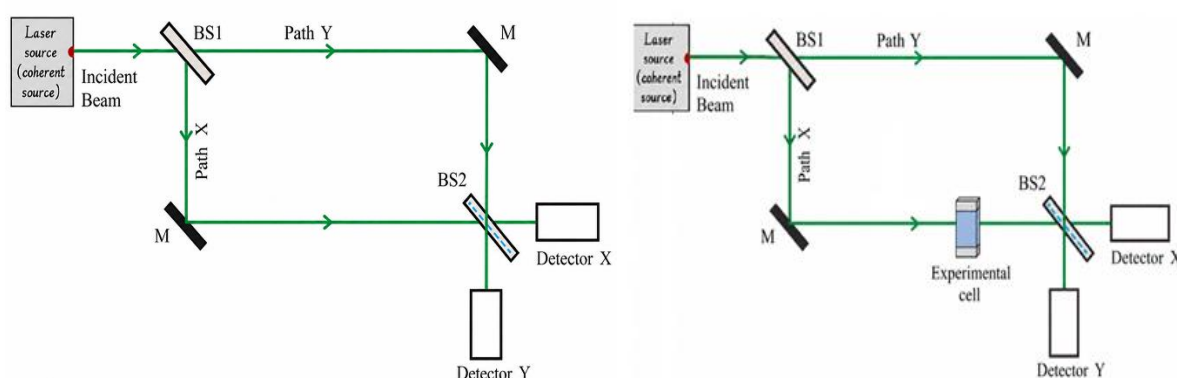
Application:

Mach–Zehnder interferometer (MZI):

A Mach–Zehnder interferometer (MZI) is an optical instrument that operates on the principle of **light interference**. By analyzing the resulting interference pattern, it can measure extremely small changes in phase, refractive index, optical path length, displacement, and other physical parameters.

Construction & Working:

The block diagram of a Mach-Zehnder interferometer is shown below. It consists of a laser source, two beam splitters, two plane mirrors, and detectors. Beam splitters are used to divide the incident light into two coherent beams of equal intensity. The mirrors will reflect the beam in different paths. An experimental cell may be used to alter the phase difference between the two beams.



The light from the laser source is split into two parts by the beam splitter BS-1. The reflected beam travels along the path X and the transmitted beam along Y. The reflected beam suffers a phase change of 180° whereas the phase of the transmitted beam remains unaltered. These two beams are incident on BS-2 after reflecting from the mirrors M and then enter the detectors X and Y.

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The beams travelling along path X and path Y arrive at the detector 'Y' with a phase difference of 180° . They interfere destructively, and no light enters detector 'Y'. The beams entering the detector 'X' will have the same phase and interfere constructively.

If an experimental sample is introduced in one of the arms, an additional optical path difference and hence phase difference is introduced, which changes the interference pattern. The interference pattern is then analyzed to measure the physical quantity responsible for the phase change. Thus, the Mach–Zehnder interferometer can be used to measure a controlled change in phase difference between two coherent light beams.

Applications:

Mach–Zehnder interferometer used

1. to determine the refractive index of gases and transparent materials
2. in quantum computing for studying quantum interference, quantum key distribution, and quantum optical circuits
3. in fiber optic sensors to measure temperature, pressure, and strain
4. in optical fiber communication as optical modulators and switching devices.

Solved Numerical Problems:

1. The refractive indices of core and cladding are 1.49 and 1.47, respectively, in an optical fiber. Find the numerical aperture and angle of acceptance.

Given: RI of Core, $n_1 = 1.49$

RI of cladding, $n_2 = 1.48$

To find: Numerical aperture, N.A = ?

Acceptance angle, $\theta = ?$

Solution: We know that the numerical aperture is given by,

$$\begin{aligned} NA &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{1.49^2 - 1.47^2} \\ &= 0.243 \end{aligned}$$

The angle of acceptance θ is related to N.A. through the equation:

$$\begin{aligned} \theta &= \sin^{-1}(\text{N.A.}) = \sin^{-1}(0.243) \\ \theta &= 14.06 \end{aligned}$$

The values of numerical aperture and angle of acceptance are 0.243 and 14.06, respectively.

2. An optical fiber has a core material with refractive index 1.57 and its cladding material has a refractive index of 1.54. The light is launched into it in air. Calculate its numerical aperture and acceptance.

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Given: RI of Core, $n_1 = 1.57$

RI of Cladding $n_2 = 1.54$

To find: Numerical aperture, N.A. ?

Acceptance angle, $\theta = ?$

Solution: We know that the numerical aperture is given by,

$$\begin{aligned} NA &= \sqrt{n_1^2 - n_2^2} \\ &= \sqrt{1.57^2 - 1.54^2} \\ &= 0.305 \end{aligned}$$

The angle of acceptance θ is related to N.A. through the equation,

$$\begin{aligned} \theta &= \sin^{-1}(\text{N.A.}) = \sin^{-1}(0.305) \\ \theta &= 17.75 \end{aligned}$$

3. An optical glass fiber of refractive index 1.48 is to be clad to ensure total internal reflection that will contain light travelling within 6° of the fiber axis. What maximum index of refraction is allowed for the cladding?

Given: RI of Core, $n_1 = 1.48$

Angle between the ray and the fiber axis = 6°

To find: RI of cladding, $n_2 = ?$

Solution: Angle of incidence of the ray at the core-cladding interface = 84°

For grazing incidence,

$$n_1 \sin 84^\circ = n_2 \sin 90^\circ$$

$$1.48 \sin 85 = n_2$$

$$n_2 = 1.47$$

$\therefore$ If $n_2 = 1.47$, the refracted ray grazes along the interface, or the ray suffers total internal reflection since $n_2 < 1.48$.

4. The N.A. of an optical fiber is 0.3 when surrounded by air. Determine the R.I. of its core given the R.I. of the cladding is 1.6. Also find the acceptance angle when the fiber is in a medium of R.I of 1.33.

Given: N.A. while the fiber is in air, $(\text{N.A.})_{\text{air}} = 0.3$

RI of Cladding, $n_2 = 1.6$

R.I. of water, $n_o = 1.33$

To find: RI of Cladding $n_1 = ?$

Acceptance angle while the fiber is in water, $\theta_o = ?$

Solution: In Air, N.A. is given by

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$$(N.A.)_{\text{air}} = \sqrt{n_1^2 - n_2^2} = 0.3$$

$$\therefore 0.3 = \sqrt{n_1^2 - 1.6^2}$$

Squaring both sides

$$0.09 = n_1^2 - 2.56$$

$$n_1^2 = 2.56 + 0.09 = 2.65$$

$$n_1 = 1.63$$

In water, N.A. is given by

$$(N.A.)_{\text{water}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \frac{(N.A.)_{\text{air}}}{1.33} = \frac{0.3}{1.33}$$

$$(N.A.)_{\text{water}} = 0.225$$

But $N.A. = \sin \theta_0$

$\therefore$ The acceptance angle,

$$\theta_0 = \sin^{-1} (N.A.) = \sin^{-1} (0.225)$$

$$\theta_0 = 13^\circ$$

5. The angle of acceptance of an optical fiber is 35° when surrounded by air. Find the acceptance when it is in the refractive index of 1.33.

Given: Angle of acceptance in air $\theta_0 = 35^\circ$

RI of refracting medium, $n'_0 = 1.33$

To find: Angle of acceptance while the fiber is in the refractive medium $\theta'_0 = 1.33$

Solution: We have the equation,

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

when the surrounding medium is air, then, $n_0 = 1$, $\theta_0 = 35^\circ$

$$\sin 35 = \frac{\sqrt{n_1^2 - n_2^2}}{1}$$

$$0.57 = \sqrt{n_1^2 - n_2^2} \dots\dots\dots (1)$$

For a medium with R.I = 1.33,

$$\sin \theta'_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

where $n_0 = 1.33$ & $0.57 = \sqrt{n_1^2 - n_2^2}$

$$\therefore \sin \theta'_0 = \frac{0.57}{1.33} \qquad \theta'_0 = 25.37^\circ$$

6. Calculate the V-number for a fiber of core-diameter $40 \mu\text{m}$ and with refractive indices of 1.55 and 1.5 respectively for core and cladding when the wavelength of

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the propagating wave is 1400 nm. Also calculate the number of modes that the fiber can support for propagation. Assume that the fiber is in air.

Solution:

Given: Core-diameter, $d = 40 \mu\text{m}$

Refractive index of core, $n_1 = 1.55$

Refractive index of cladding, $n_2 = 1.5$

Wavelength of light, $\lambda = 1400 \text{ nm}$

To find: V-number for a fiber, $V = ?$

Number of modes, $M_n = ?$

We have, $V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$

$$V = \frac{3.14 \times 40 \times 10^{-6}}{1400 \times 10^{-9}} \left(\sqrt{(1.55)^2 - (1.5)^2} \right)$$

$$V = 35$$

$$\text{Number of modes, } M_n = \frac{V^2}{2} = \frac{35^2}{2} = 612.5$$

7. The angle of acceptance of an optical fiber is 30° when kept in air. Find the angle of acceptance when it is in a medium of refractive index 1.33.

Solution:

Given: Acceptance angle in air, $\theta_{0a} = 30^\circ$

Refractive index of surrounding medium, $n_{0m} = 1.33$

To find: Acceptance angle in air, $\theta_{0m} = ?$

Using the equation,

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

When fiber is in air,

$$\sin \theta_{0a} = \sqrt{n_1^2 - n_2^2} \text{ --- (1)}$$

When fiber is in medium,

$$\sin \theta_{0m} = \frac{\sqrt{n_1^2 - n_2^2}}{n_{0m}}$$

Substituting From (1)

$$\sin \theta_{0m} = \frac{\sin \theta_{0a}}{n_{0m}}$$

$$\sin \theta_{0m} = \frac{\sin 30}{1.33}$$

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$$\sin \theta_{om} = \frac{0.5}{1.33} = 0.3759$$

$$\theta_0 = \sin^{-1}(0.3759)$$

$$\theta_0 = 22^\circ$$

Sample Questions

1. Define numerical aperture and acceptance angle of an optical fiber and hence derive an expression for the numerical aperture of the fiber in air. (L2)
2. Define attenuation in optical fibers and explain the various attenuation mechanisms with suitable examples. (L2)
3. Describe the construction and working of a Mach–Zehnder interferometer and mention its applications. (L2)
4. An optical fiber has a core material with RI 1.55, and its cladding material has RI 1.50. The light is launched into it in air. Calculate its N.A. and acceptance angle. (L3)
