

MODULE – 3: Mechanical Properties of Materials and Applications

Introduction, Elastic Moduli, Poisson's ratio, Relation between Y , η , and σ , Relation between K , Y , and σ , Limiting Values of Poisson's ratio. Bending Moment, Young's Modulus using Uniform Bending, Failure of Engineering Materials – Ductile Fracture, Brittle Fracture, Stress Concentration, & Fatigue, Shape Memory Alloys.

Introduction:

The study of strength of materials is to provide the means of analysing and designing various machines and load bearing structures. Elasticity is an elegant and fascinating subject that deals with determination of the stress, strain and displacement distribution in an elastic solid under the influence of external forces. Following the assumptions of linear, small deformation theory, the formulation establishes a mathematical model providing solutions to problems that have applications in many engineering and scientific fields. Civil engineering applications include stress and deflection analysis of structures like rods, beams, plates, shells soil, rock concrete etc. Mechanical engineering uses elasticity in numerous problems of thermal stress analysis, fracture mechanics, fatigue, and design of machine elements. Material engineering uses elasticity to determine the stress fields of crystalline solids, dislocations, microstructures etc. Applications in aeronautical engineering include stress fluctuations, fracture, and fatigue analysis in aero structures. The subject also provides the basis for study of materials behaviour in plasticity and viscoelasticity.

Elasticity:

*The property of material body to regain its original shape and size on removal of the deforming forces is called **elasticity**.*

Within certain elastic limit steel and quartz show elastic properties. The elastic property is desirable for materials used in tools and machines.

Importance of Elasticity in Engineering Applications:

A sound knowledge of elastic properties of materials is very essential in the field of engineering. Engineers can make better choice of the materials for their use by knowing the nature of its stress-strain curve. Pure metals are soft by property. They are ductile and have low tensile strength. Hence they are rarely used in engineering applications. Alloys are generally harder than

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pure metals. They are produced by blending (mixing) different metals after which, they exhibit unique properties that are different from metals mixed. These alloys offer better elastic properties useful for engineering applications.

Plasticity:

A body which does not show any tendency to recover their original condition are said to be Plastic and the property is called plasticity.

E.g. – polyethylene, Polystyrene etc.

This property of the material is necessary for forging, stamping images on coins and ornamental works.

Load:

The term load implies the combination of external forces acting on a body and its effect is to change the form or the dimensions of the body. It is essentially a deforming force.

Stress:

When the deforming forces acting on a body, the restoring or recovering force per unit area of cross section set up inside the body is called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Cross sectional area}} = \frac{F}{A} \text{ N/m}^2$$

Types of stress:

1. Tensile Stress:

When a body or section is subjected to two equal and opposite pulls and if it tends to pull apart the particles of the material causing extension in the direction of application of load, then the load is called **tensile load** and the corresponding stress induced is known as **tensile stress**. *Longitudinal stress or tensile stress is applied along the length and hence causes change in length.*

2. Compressive stress (Pressure or volume stress):

When a body or section is subjected to two equal and opposite pushes and it tends to push the particles of the material nearer causing shortening in the direction of application of load,

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then the load is called **compressive load** and the corresponding stress induced is known as **compressive stress**.

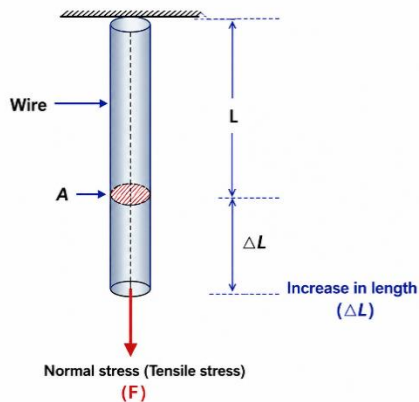
3. Tangential stress or shearing stress:

When a body is subjected to a force acting along the tangential direction, the body experiences a turning or twisting effect resulting in the change in the shape of the body without any change in its volume, the stress induced is called **shearing stress** or **tangential stress**.

Strain:

When a body is subjected to external force, there will be change in dimensions of the body. The change in dimension is called deformation.

The ratio of change in dimension of body or deformation to the original dimension of the body is called known as strain.



If a bar is subjected to a direct load and hence a stress, the bar will change in length. If the bar has an original length L and changes by an amount ΔL , the strain produced is defined as follows:

$$\text{Strain} = \frac{\text{change in dimension of the body}}{\text{original dimension}} = \frac{L}{\Delta L}$$

Strain is thus, a measure of the deformation of the material and is a non-dimensional Quantity i.e. it has no units.

Since in practice, the extensions of materials under load are very small, it is often convenient to measure the strain in the form of strain $\times 10^{-6}$ i.e., micro strain.

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Types of strain:

1. Linear or Longitudinal Strain:

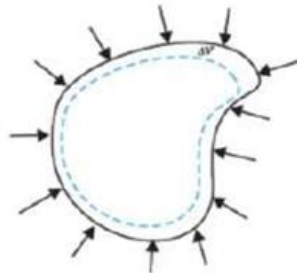
When a force is applied, the ratio of change in the length to the original length of a body is known as longitudinal strain.

$$\text{Longitudinal Strain} = \frac{\text{change in length of the body}}{\text{original length}} = \frac{\Delta L}{L}$$

2. Volume strain:

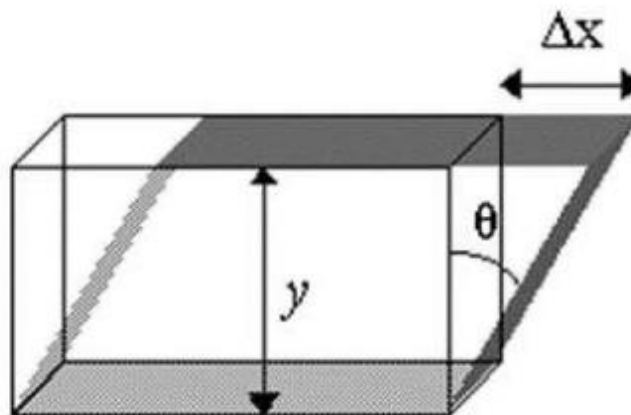
When uniform pressure is applied normally on all over the surface of a body, the body undergoes a change in its volume. The ratio of change in the volume to the original volume is called as volume strain.

$$\text{Volume strain} = \frac{\Delta V}{V}$$



3. Shear Strain:

When a body is subjected to tangential force, the angular displacement of a reference line in the body is as known shear strain. The shearing angle itself is the measure of the ratio of change in dimension to the original dimension.



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$$\text{Shearing strain} = \tan\theta = \Delta x / y$$

When the angle of shear is small,

$$\text{Shearing strain} = \theta = \Delta x / y$$

Hooke's law

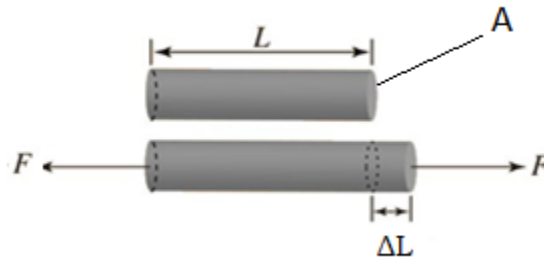
Hooke's law states that when a material is loaded within its elastic limit, **stress is directly proportional strain**.

It means that the ratio of stress to strain is constant within the elastic limit. This constant is known as Modulus of elasticity.

$$\frac{\text{stress}}{\text{strain}} = \text{constant or modulus of elasticity}$$

Types of Elastic Moduli:

1. Young's Modulus of Elasticity (Y):



When a wire is acted upon by two equal and opposite forces in the direction of its length, the length of the body is changed. The change in length per unit length ($\Delta L/L$) is called the longitudinal strain and the restoring force (which is equal to the applied force in equilibrium) per unit area of cross-section of wire is called the longitudinal stress.

For small change in the length of the wire, the ratio of the longitudinal stress to the corresponding strain is called the **Young's modulus of elasticity (Y)** of the wire.

$$\text{Thus, } Y = \text{Longitudinal stress} / \text{Linear Strain} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

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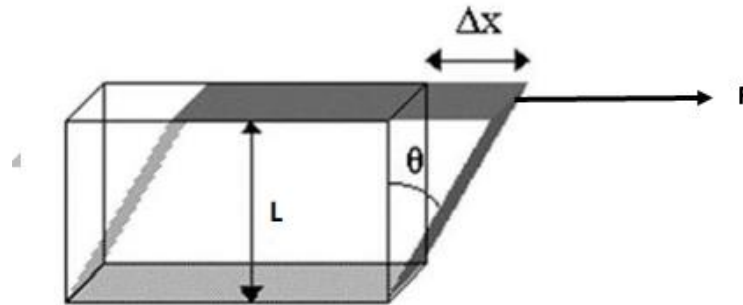
Let there be a wire of length 'L' and radius 'r'. Its one end is clamped to a rigid support and a mass M is attached at the other end. Then

$$F = Mg \text{ and } A = \pi r^2$$

Substituting in above equation, we have,

$$Y = \frac{MgL}{(\pi r^2)\Delta L}$$

2. Modulus of Rigidity (n or η):



When a body is acted upon by an external force tangential to a surface of the body, the opposite surfaces being kept fixed, it suffers a change in shape of the body, and its volume remains unchanged. Then the body is said to be sheared. The tangential force acting per unit area of the surface is called the '**shearing stress**' (F/A).

The ratio of displacement to perpendicular distance between the two surfaces is known as **shearing strain (θ)**.

Shearing strain $\theta = \frac{\Delta x}{L}$ when θ is small.

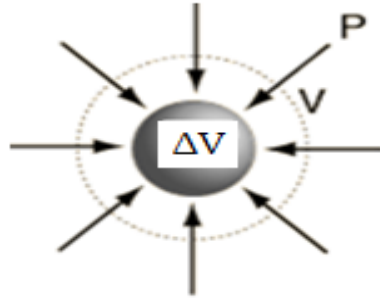
For small strain, the ratio of the shearing stress to the shearing strain is called the '**modulus of rigidity**' of the material of the body. It is denoted by '**n or η** '.

Rigidity modulus (n or η) = Tangential stress / shear Strain

$$n = \frac{F/A}{\Delta x/L} = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

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3. Bulk Modulus of Elasticity (K)



Describes volumetric elasticity or the tendency of an object to deform in all directions when uniformly loaded in all directions. It is defined as volumetric stress over volumetric strain, and is the inverse of compressibility.

$$K = \frac{\text{Volumetric stress}}{\text{Volume strain}} = \frac{PV}{\Delta V}$$

When a uniform pressure (normal force) is applied all over the surface of a body, the volume of the body changes. The change in volume per unit volume of the body is called the ‘**volume strain**’ and the normal force acting per unit area of the surface (**pressure**) is called the normal stress or **volume stress**.

For small strains, the ratio of the volume stress to the volume strain is called the ‘**Bulk modulus of the material of the body**’. It is denoted by K.

$$\text{Then, } K = \frac{-P}{\frac{\Delta V}{V}}$$

Negative sign in formula implies that when the pressure increases volume decreases and vice-versa.

The reciprocal of the Bulk modulus of the material of a body is called the “**compressibility**” of that material. Thus,

$$\text{Compressibility} = 1/K$$

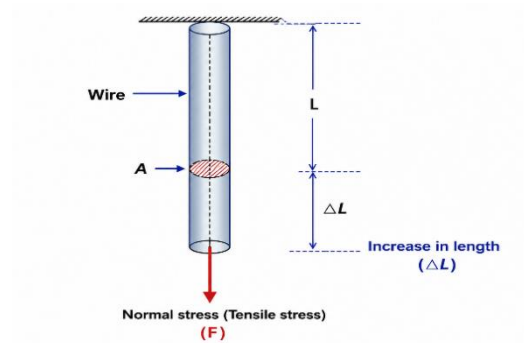
Longitudinal Strain Coefficient (α):

The longitudinal strain produced per unit stress is called longitudinal strain coefficient.

$$\alpha = \frac{\text{longitudinal strain}}{\text{applied stress}}$$

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$$= \frac{\Delta L/L}{T} = \frac{\Delta L}{TL}$$



The extension produced due to the applied stress 'T' is $\Delta L = TL\alpha$

Lateral Deformation:

When there is a Longitudinal Strain in a material due to the deforming forces acting along the length, there is always a change in the thickness or diameter of the material. This change occurs in a direction perpendicular to the direction of the deforming force and is called lateral change.

Lateral stain:

If a deforming force acting on a wire of circular cross section with the original diameter 'D', produces a change 'd' in its diameter then,

$$\text{Lateral strain} = \frac{d}{D}$$

Lateral Strain Coefficient (β):

The lateral strain produced per unit stress is called lateral strain coefficient.

$$\beta = \frac{\text{lateral strain}}{\text{applied stress}}$$

$$\beta = \frac{d/D}{T} = \frac{d}{TD}$$

Poisson's ratio (σ):

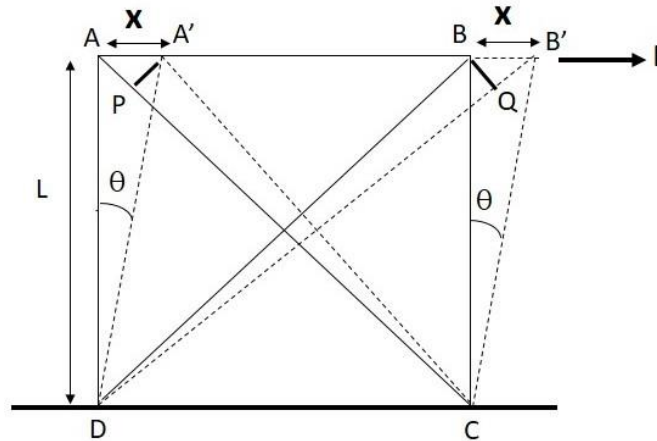
When a material is stretched, the increase in its length (Linear strain α) is accompanied by decrease in cross section (lateral strain β). Within the elastic limit, the lateral strain is

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proportional to longitudinal strain and the ratio between them is a constant for a material known as **Poisson ratio (σ)**.

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} \text{ i.e., } \sigma = \frac{\beta}{\alpha}$$

Expression for Young's Modulus (Y), Rigidity Modulus (η) and Poisson's Ratio (σ) :



Consider a cube of side 'L', whose lower surface CD is fixed to a rigid support. Let a tangential force 'F' is applied at the upper surface along AB of the cube in a direction as shown in figure. The applied tangential force causes the relative displacements at different parts of the cube, so that, A moves to A' and B moves to B' through a small angle ' θ '. Due to this, the diagonal AC will be shortened to A'C and diagonal DB will be increased to a length DB'. We have an extension Strain along DB and compression strain along AC. The angle ' θ ' is the angle of shear (shearing strain) which is very small in magnitude.

$$\text{The shearing stress acting on the body, } T = \frac{F}{A} = \frac{F}{L^2}$$

$$\text{From figure, shearing strain, } \theta = \frac{x}{L} \text{ ----- (1)}$$

Shearing stress along AB is equivalent to elongation stress along BD and compressive stress along AC. Let α be the longitudinal strain coefficient and β be the lateral strain coefficient respectively. Then

$$\text{Elongation along BD due to the tensile stress along BD} = BD.T.\alpha$$

$$\text{Elongation along BD due to the compressive stress along AC} = BD.T.\beta$$

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Total extension along BD is

$$QB' = BD \cdot T (\alpha + \beta) = \sqrt{2}LT (\alpha + \beta) \quad \text{---- (2) [since } BD = \sqrt{2}L]$$

Also, from right angled triangle QBB',

$$QB' = BB' \cos 45^\circ$$

$$QB' = \frac{x}{\sqrt{2}} \quad \text{----- (3)}$$

Eqn (2) becomes, $\frac{x}{\sqrt{2}} = \sqrt{2}LT (\alpha + \beta)$

$$\frac{x}{LT} = 2 (\alpha + \beta)$$

$$\frac{\theta}{T} = 2 (\alpha + \beta) \quad \text{(Since from Eqn. (1))}$$

Taking the reciprocal,

$$\frac{T}{\theta} = \frac{1}{2(\alpha + \beta)}$$

But $\frac{T}{\theta} = \eta$, the rigidity modulus

Therefore, $\eta = \frac{1}{2(\alpha + \beta)} \quad \text{----- (4)}$

Young's modulus, $Y = \frac{\text{Longitudinal Stress}}{\text{Linear strain}}$

$$= \frac{1}{\frac{\text{Linear Strain}}{\text{Longitudinal Stress}}} = \frac{1}{\alpha}$$

Therefore, $Y = \frac{1}{\alpha} \quad \text{----- (5)}$

This is the relation between young's modulus (Y) and linear strain (α).

We have from (4); $2(\alpha + \beta) = \frac{1}{\eta}$

$$2\alpha \left(1 + \frac{\beta}{\alpha}\right) = \frac{1}{\eta}$$

$$2\eta(1 + \sigma) = \frac{1}{\alpha} \left(\because \frac{\beta}{\alpha} = \sigma\right)$$

$$2\eta(1 + \sigma) = Y \left(\because \frac{1}{\alpha} = Y\right)$$

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The equation $((1 + \sigma) = Y)$ gives relationship between Young's Modulus (Y), Rigidity Modulus (n) and Poisson's Ratio (σ)

Relationship between Bulk Modulus (K), Young's Modulus (Y), and Poisson's Ratio (σ):

$$K = \frac{Y}{3(1 - 2\sigma)}$$

$$Y = 3K(1 - 2\sigma)$$

Limits of Poisson's ratio σ :

We have

$$Y = 3K(1 - 2\sigma) \text{ and } Y = 2n(1 + \sigma)$$

Therefore, $3K(1 - 2\sigma) = 2n(1 + \sigma)$

1. If σ be a positive quantity, $(1 - 2\sigma)$ should be positive

$$2\sigma < 1 \quad \Rightarrow \quad \sigma < 0.5,$$

A perfect incompressible material deformed elastically at small strains would have a Poisson's ratio exactly 0.5.

If σ be a negative quantity, $(1 + \sigma)$ should be positive. This implies that $\sigma > -1$

Thus the limiting values of σ lies between -1 and 0.5. Negative value of σ would mean that on being extended, a body should also expand laterally. This hardly happens ordinarily. Similarly a value of $\sigma = 0.5$ would mean that substance is perfectly incompressible. Generally the limiting values of Poisson's ratio of different materials varies from -1 to 0.5

BENDING OF BEAM:

A homogenous body of uniform cross section whose length is large compared to its other dimensions is called a beam.

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Neutral Surface and Neutral Axis:

Consider a uniform beam MN whose one end is fixed at M (Fig a). The beam can be thought of as made up of a number of parallel layers and each layer in turn as made up of a number of thin parallel longitudinal filaments or fibers in the plane of the layer. If a cross section of the beam along its length and perpendicular to these layers is taken the filaments of different layers appear like straight lines piled one above the other along the length of the beam (Fig b).

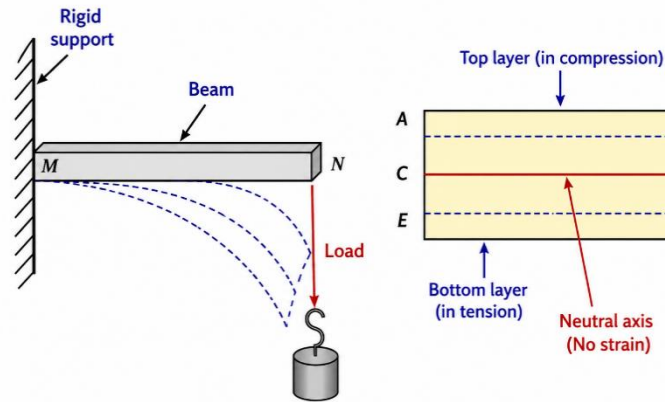
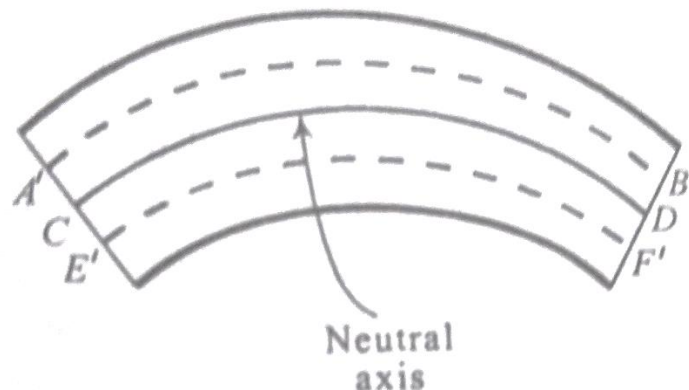


Fig (a)

Fig (b)

If a load is attached to the free end of the beam, the beam bends. The successive layers along with constituent filaments are strained. A filament like AB of an upper layer will be elongated to A^1B^1 and the one like EF of a lower layer will be compressed to E^1F^1 . The layer (CD) that does not undergo any change in the dimension is called as the **neutral layer** or **neutral surface**. A filament in the neutral layer whose length always remains the same is called the **neutral axis**.



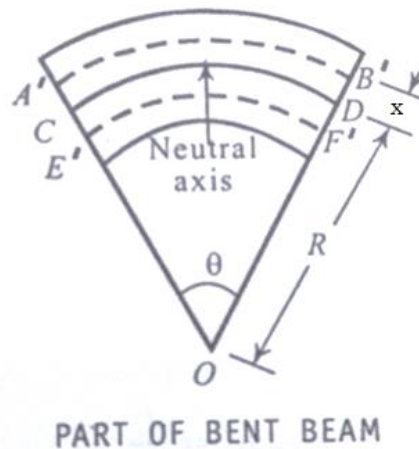
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Neutral surface: is that layer of a uniform beam which does not undergo any change in its dimension when the beam is subjected to bending within the elastic limit.

Neutral Axis is an axis in the cross section of a beam along which there are no longitudinal stresses or strain. The length of this axis remains the same when the beam is subjected to bending.

Expression for bending moment of a beam:

Consider a uniform beam fixed at one end and loaded at the other. As a result an equal reaction force acts in the upward direction at the fixed end. These two equal and opposite forces constitute a couple known as bending couple due to which the beam bends. The moment (rotating effect) of the bending couple due to which a beam undergoes bending is called the **bending moment**.



Due to the bending, a layer like AB above the neutral layer will be elongated to $A'B'$ and the one like EF below the neutral layer will be compressed to $E'F'$. CD is the neutral surface which does not undergo any change in its length. The layers of the bent beam forms the part of concentric circles with centre at O. Let 'R' be the radius of the circle to which the neutral surface forms a part. Let ' θ ' be the angle subtended by the layers at the common centre 'O' and 'x' be the separation between the successive layers.

The change in length of layer AB is $= A'B' - CD$

But $CD = R\theta$ (From fig.)

If 'x' is the separation between the successive layers then from figure

$$A'B' = (R + x)\theta$$

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$$\begin{aligned}\text{Change in length} &= A'B' - CD \\ &= (R + x)\theta - R\theta \\ &= x\theta\end{aligned}$$

$$\text{Linear strain} = \frac{\text{change in length}}{\text{original length}} = \frac{x\theta}{R\theta} = \frac{x}{R} \text{----- (1)}$$

$$\text{Young's modulus, } Y = \frac{\text{Longitudinal stress}}{\text{Linear strain}}$$

$$\begin{aligned}\text{Longitudinal stress} &= Y \times \text{Linear Strain} \\ &= Y \times \frac{x}{R} \text{----- (2)}\end{aligned}$$

$$\text{But} \quad \text{Longitudinal stress} = \frac{F}{a}$$

Where F is the force acting on the beam and 'a' is the area of the layer 'AB'.

$$\text{Therefore } \frac{F}{a} = Y \times \frac{x}{R}$$

$$\text{Or} \quad F = \frac{Yxa}{R}$$

The moment of force = Force $\times$ distance of AB from neutral axis

$$= F \times x = \frac{Yax^2}{R} \text{----- (3)}$$

$$\text{Moment of the force acting on the entire layer} = \sum \frac{Yax^2}{R}$$

$$= \frac{Y}{R} \sum ax^2$$

Here $\sum ax^2$ is called the **geometric moment of Inertia** I_g

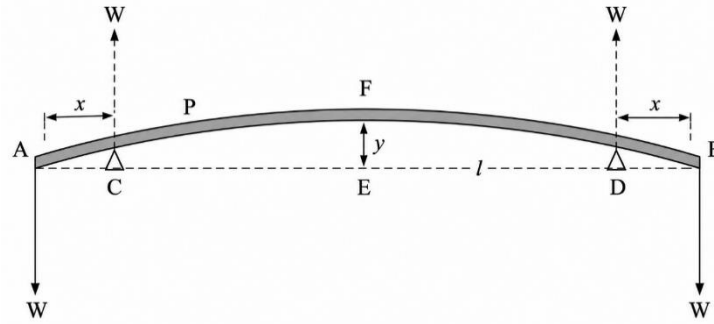
i.e. $I_g = \sum ax^2 = AK^2$; where K is called the radius of gyration about the neutral axis.

$$\text{Therefore } \textbf{Moment of the force or bending moment} = \frac{Y}{R} I_g$$

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Young's modulus by uniform bending:

Consider a rectangular beam AB of negligible mass placed symmetrically on two knife edges C and D in a horizontal level. Let $CD = l$. Equal loads W are suspended at two ends of the beam each at a distance x from the respective knife edge, such that $AC = BD = x$.



Then the beam bends and the centre of the beam is elevated from E to F. Let $EF = y$ be the elevation. Consider the cross-section of the beam at the point P.

The only forces acting on the part PA of the beam are

- (i) the force W at A and
- (ii) the reaction W at C

Anticlockwise moment of the load W at A about P = $W \times AP$. This moment is taken as positive. Clockwise moment of the reaction W at C about P = $W \times CP$. This moment taken as negative.

$$\begin{aligned} \text{External bending moment} &= W \times AP - W \times CP \\ &= W \times (AP - CP) = W \times AC = W \times x \end{aligned}$$

This must be balanced by internal bending moment for equilibrium.

$$\text{Internal bending moment} = \frac{YI}{R}$$

Where Y is Young's modulus, I_g is geometrical moment of inertia and R is radius of curvature of the arc CP.

For equilibrium, External bending moment = Internal bending moment

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$$W \times x = \frac{Y I_g}{R} \quad \dots(1)$$

For a given load W , x , Y and I_g are constants. Therefore, R is also a constant. Hence, the beam is bent into an arc of a circle with same radius of curvature R at all points. The bending is then said to be uniform.

Using the property of circle

$$CE \times ED = FE \times EG$$

$$\frac{l}{2} \times \frac{l}{2} = y \times (2R - y)$$

$$\frac{l^2}{4} = 2yR$$

y^2 is neglected in comparison with $2R$

$$\therefore y = \frac{l^2}{8R} \quad \dots(2)$$

From eqn (1)

$$R = \frac{Y I_g}{W x}$$

Substituting the value of R in eqn (2),

$$y = \frac{W x l^2}{8 Y I_g} \quad \dots(3)$$

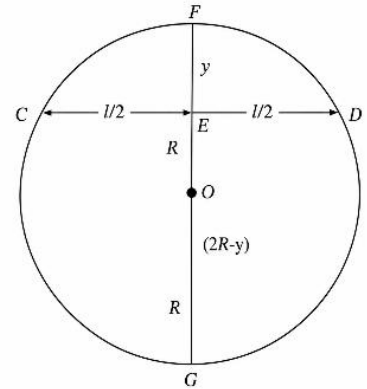
$$\therefore Y = \frac{W x l^2}{8 y I_g} \quad \dots(4)$$

For a rectangular beam,

$$I_g = \frac{b d^3}{12} \text{ Where } b \text{ and } d \text{ are the breadth and thickness of the beam respectively.}$$

Substituting the value of I_g and $W=mg$ in eqn. (4),

$$Y = \frac{3 m g x l^2}{2 b d^3 y}$$



Property of the circle

Failure of Engineering Materials:

Failure of a material is the loss of its ability to perform the intended function due to excessive stress, deformation, or repeated loading. Material failure may occur suddenly or gradually depending on the nature of the material and the type of loading.

The common modes of failure are: (i) Ductile fracture (ii) Brittle fracture (iii) Stress concentration (iv) Fatigue

1. *Ductile Fracture*

Ductile fracture is the failure of a material accompanied by noticeable plastic deformation.

Characteristics

Large plastic deformation before fracture.

Crack propagates slowly.

High energy absorption.

2. *Brittle Fracture*

Brittle fracture is the sudden failure of a material with little or no plastic deformation.

Characteristics

Negligible plastic deformation.

Rapid crack propagation.

Low energy absorption.

3. *Stress Concentration*

Stress concentration is the localized increase in stress around discontinuities such as holes, notches, sharp corners, grooves or cracks.

$$\text{Stress Concentration Factor } K_t = \frac{\text{Nominal Stress}}{\text{Maximum Stress}}$$

The stress concentration factor is used to determine the increase in local stress near discontinuities and to design engineering components that are safe against fracture and fatigue.

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4. Fatigue

Fatigue is the progressive failure of a material due to repeated or cyclic loading, even when the applied stress is below the yield strength.

Factors Affecting Fatigue are magnitude of cyclic stress, Stress concentration, Surface roughness, corrosion, temperature and material properties

Shape Memory Alloys:

Shape Memory Alloys (SMAs) are a unique class of metallic alloys that have the ability to recover their original shape when heated above a certain temperature after being deformed..

The temperature or temperature range over which an SMA transforms from its deformed shape back to its original shape is called the transformation temperature

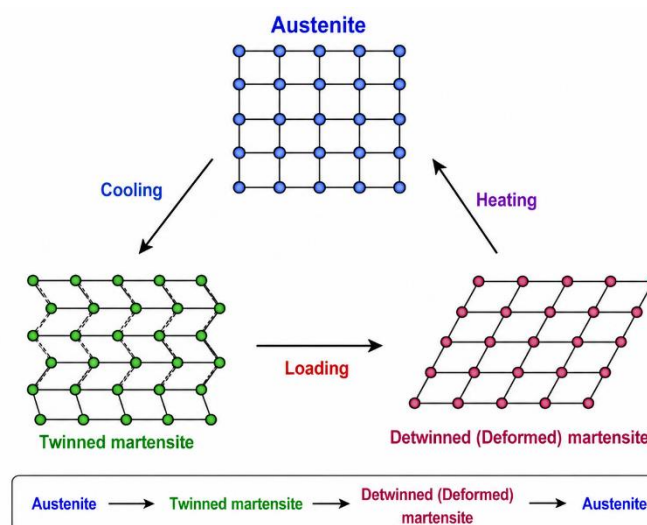
Types of SMAs

There are two types of SMAs

- (i) *One way shape memory alloys* – Materials take up their own shape only upon heating
- (ii) *Two way shape memory alloys* – Materials take up their own shape not only upon heating and also upon cooling

Characteristic of SMA

SMA has two stable phases - the high-temperature phase, called *austenite* and the low-temperature phase, called *martensite*.



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Shape Memory Effect: The ability of a shape memory alloy to recover its original shape upon heating after being deformed.

Pseudoelasticity (Superelasticity): The ability of a shape memory alloy to undergo large deformation and recover its original shape immediately upon unloading, without heating.

Properties

- (1) It has less weight but having high strength
- (2) They have good mechanical properties
- (3) It has high corrosion resistance
- (4) They are bio-compatible

Applications of shape memory alloys

Eye glass frames, Helicopter blades, Coffee Valves

Solved Numerical Problems:

1. In a stretching experiment, the extension produced in a wire for a load of 14.7 N is 0.25 cm, the length of the wire is 1.8 m and its radius is 0.12 cm. Find the Young's modulus of the material of the wire.

Given:

$$F = 14.7 \text{ N}$$

$$\text{Change in length } \Delta L = 0.25 \text{ cm} = 0.25 \times 10^{-2} \text{ m}$$

$$\text{Length of the wire } L = 1.8 \text{ m}$$

$$\text{Radius } r = 0.12 \text{ cm} = 0.12 \times 10^{-2} \text{ m}$$

$$\text{Area } A = \pi r^2$$

$$\text{Formula: } Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$Y = \frac{14.7 \times 1.8}{3.14 \times (0.12 \times 10^{-2})^2 \times 0.25 \times 10^{-2}}$$

$$Y = 2.34 \times 10^9 \text{ N/m}^2 \text{ or } Y = 2.34 \text{ GPa}$$

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2. Calculate the Poisson's ratio for silver, given its Young's modulus is $7.25 \times 10^{10} \text{ N/m}^2$ and rigidity modulus is $2.7 \times 10^{10} \text{ N/m}^2$

Given

$$Y = 7.25 \times 10^{10} \text{ N/m}^2$$

$$\eta = 2.7 \times 10^{10} \text{ N/m}^2$$

Formula

$$Y = 2\eta(1 + \sigma)$$

$$\sigma = \frac{Y}{2\eta} - 1$$

$$\sigma = \frac{7.25 \times 10^{10}}{2 \times 2.7 \times 10^{10}} - 1$$

$$\sigma = 0.3426$$

3. A rectangular beam is subjected to uniform bending. Under a certain load, the elevation at the midpoint of the beam is 10 mm. Determine the elevation produced under the same load if another beam of the same material has twice the distance between the knife edges, twice the breadth, and three times the thickness. Assume the distance of the load from the knife edge remains unchanged.

Formula

$$Y = \frac{3mgx l^2}{2bd^3y}$$

Here load applied is same and breadth, thickness of the beam is also same in two experiment. Since same material used in the experiments hence young's modulus is same.

$$\text{The elevation } y \propto \frac{l^2}{bd^3}$$

$$\text{For 1}^{\text{st}} \text{ experiment } y_1 \propto \frac{l_1^2}{b_1 d_1^3} \text{ and for 2}^{\text{nd}} \text{ experiment } y_2 \propto \frac{l_2^2}{b_2 d_2^3}$$

$$\frac{y_2}{y_1} = \frac{l_2^2}{b_2 d_2^3} \times \frac{b_1 d_1^3}{l_1^2}$$

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Here $l_2 = 2l_1$, $b_2 = 2b_1$ and $d_2 = 3d_1$

$$\frac{y_2}{y_1} = \frac{l_2^2}{b_2 d_2^3} \times \frac{b_1 d_1^3}{l_1^2}$$

$$\frac{y_2}{y_1} = \frac{4l_1^2}{2b_1 \times 27d_1^3} \times \frac{b_1 d_1^3}{l_1^2} = \frac{2}{27}$$

And $y_1 = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$

So

$$y_1 = \frac{2}{27} y_2 = \frac{2}{27} \times 10 \times 10^{-3}$$

$$y_1 = 0.74 \text{ mm}$$

4. A rectangular beam of breadth 3 cm and thickness 0.5 cm is subjected to uniform bending. The distance between the knife edges is 80 cm and the distance of each load from the nearest knife edge is 20 cm. Two loads of 1 kg each are suspended symmetrically at the ends, producing an elevation of 0.4 cm at the midpoint. Calculate the Young's modulus of the material of the beam.

Given

Breadth $b = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$

Thickness $d = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$

Length $l = 80 \text{ cm} = 80 \times 10^{-2} \text{ m}$

$x = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$

Mass $m = 1 \text{ kg}$

Elevation $y = 0.4 \text{ cm} = 0.4 \times 10^{-2} \text{ m}$

$$Y = \frac{3mgx l^2}{2bd^3y}$$

$$Y = \frac{3 \times 1 \times 9.8 \times 20 \times 10^{-2} \times ((80 \times 10^{-2})^2)}{2 \times 3 \times 10^{-2} (0.5 \times 10^{-2})^3 \times 0.4 \times 10^{-2}}$$

$$Y = 1.25 \times 10^{11} \text{ N/m}^2$$

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Sample Questions:

1. Derive the relation between Bulk Modulus (K), Young's Modulus (Y), and Poisson's Ratio (σ). State the limiting values of Poisson's ratio. (L2)
2. Describe a single cantilever and derive the expression for Young's modulus of a rectangular beam using the cantilever method. (L2)
3. Derive the expression for Young's modulus of a rectangular beam using the uniform bending method. (L2)
4. Derive the expression for the bending moment of a beam. (L2)
5. Explain the failure of engineering materials, highlighting ductile fracture, brittle fracture, stress concentration, and fatigue. (L2)
6. Explain the concept of Shape Memory Alloys (SMAs). Discuss their types, properties and engineering applications. (L2)