

MODULE – 4: Electrical Properties of Materials

Conductors: Quantum Free Electron Theory of Metals, Fermi-energy - Fermi Factor - Variation of Fermi Factor with Temperature, Expression for the Electrical Conductivity Based on QFET,.

Semiconductors: Expression for the Electron and Hole Concentrations, Expression for Electrical Conductivity in a Semiconductor, Hall Effect, Expression for Hall Coefficient.

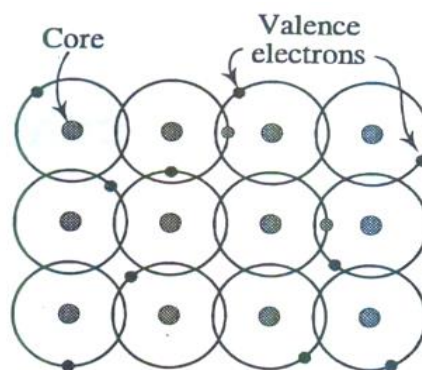
Dielectrics: Polarization and its Mechanisms. Internal field in Solids, Clausius –Mossotti Equation.

CONDUCTORS

Free-electron concept:

Metal is made up of atoms and these atoms consist of valence electrons, which are responsible for electrical conduction in the bulk state of the metal. For e.g.: A copper atom consists of 29 electrons out of which 28 electrons are bound in the 3 shells forms the core and one electron remaining in the fourth shell is the free valence electron. As a whole an atom is neutral.

When a large number of copper atoms join to form a metal the boundaries of the neighboring atoms slightly overlap on each other. Due to this, the valence electrons find continuity from atom to atom and can move easily throughout the body of the metal, but are restricted within the boundaries of the solid.



Since each atom contributes equal number of electron, there will be a very large number of electrons which are free in a metal. Such electrons are called as free electrons and they account for the bulk properties of the metal such as electrical conductivity, thermal conductivity etc. They are also called conduction electrons.

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The disconnection of the valence electrons results in a virtual loss of a negative charge for that atom so that it becomes a positive ion. The array of such ions forms a three-dimensional structure called a lattice.

Under thermal equilibrium conditions, the free electrons in a solid are in a state of random motion. At temperature T , they possess an average kinetic energy given by $\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$.

Where, v_{th} is the mean thermal velocity.

Quantum Free Electron Theory of Metals:

In 1928, Arnold Sommerfeld succeeded in overcoming many of the drawbacks of the classical free electron theory by using Pauli's exclusion principle and Fermi-Dirac statistics known as quantum free electron theory. The following are the assumptions of quantum free electron theory.

The main assumptions of quantum free electron theory are:

1. The energy values of the conduction electrons are quantized. They can have only discrete energy values.
2. The distribution of electrons in the various allowed levels occur as per Pauli's exclusion principle which states that no two electrons can have same set of quantum numbers.
3. The distribution of energy among the free electrons follows Fermi-Dirac statistics.
4. The electrons travel in a constant potential inside the metal but stay confined within its boundaries.
5. The attraction between the electrons and the lattice ions and the repulsion between the electrons are ignored.

Energy bands in solids

- ◆ In a solid, the discrete energy levels of atoms combine to form energy bands. Electrons can occupy only these allowed energy bands, which are separated by a forbidden energy gap (band gap).
- ◆ Each energy band can hold only a limited number of electrons.
- ◆ The highest occupied band is the valence band, and the band above it is the conduction band.
- ◆ Electrons gain energy to move from the valence band to the conduction band, enabling electrical conduction.

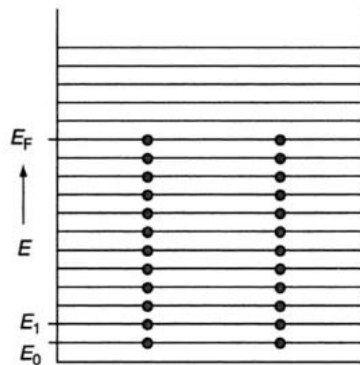
Fermi Energy:

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There are ' N ' allowed energy levels for free electrons in a metal which are separated by energy differences that are the characteristics of the material and are quantized. As per Pauli's exclusion principle, each allowed energy level can accommodate a maximum of two electrons with opposite spin. The filling up of the electrons in the energy levels should be undertaken from the lowest energy level. So, in each energy level there are two electrons one with spin up and other with spin down. However there are still higher energy levels left vacant.

"The energy of the highest occupied energy level at zero degree absolute is called Fermi energy (E_F) and the energy level is referred as Fermi level"

At absolute zero temperature i.e. when the metal is not under the influence of any electrical field or thermal energy, no electrons will be present above the Fermi level. They are completely empty and those levels below Fermi level are completely filled. Thus it indicates the electrical conductivity of a material by showing the availability of charge carriers.



Distribution of electrons in various energy levels at 0 K

Fermi Factor:

At temperatures above absolute zero, the material will be receiving thermal energy from the surroundings. So the electrons just below the Fermi level absorb the thermal energy and will occupy the higher empty levels. The electrons occupying energy levels far below the Fermi level cannot absorb this energy because there are no unoccupied higher energy levels into which the electrons can come into, when their energies increase by small amount.

Though such excitation seem to be random the resulting distribution of electrons in various energy levels after excitation will be systematic. The distribution is governed by a statistical function when the system is in thermal equilibrium (steady state).

The probability ' $f(E)$ ' that a given energy state with energy ' E ' is occupied at a steady temperature is given by

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$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

Here $f(E)$ is called Fermi factor.

“Fermi factor is defined as the probability of occupation of a given energy state for a material in thermal equilibrium”.

Dependence of Fermi factor on temperature and effect on the occupancy of energy levels:

The probability ‘ $f(E)$ ’ that a given energy state with energy ‘ E ’ is occupied at a steady temperature is given by

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

The different cases of distribution are as follows;

CASE 1: Probability of occupation for $E < E_F$ at $T = 0K$

When $T = 0K$ and $E < E_F$, we have

$$f(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1} = 1$$

$$\therefore f(E) = 1, \text{ for } E < E_F$$

It means that the energy level is certainly occupied and $E < E_F$ applies to all the energy levels below ‘ E_F ’.

CASE 2: Probability of occupation for $E > E_F$ at $T = 0K$

When $T = 0K$ and $E > E_F$, we have

$$f(E) = \frac{1}{e^{+\infty} + 1} = \frac{1}{\infty + 1} = 0$$

$$\therefore f(E) = 0, \text{ for } E > E_F$$

$\therefore$ At $T = 0K$, all the energy levels above the Fermi level are unoccupied.

Thus, at $T = 0K$, the variation of $f(E)$ for different energy values, becomes a step function as shown in the figure below

CASE 3: Probability of occupation at ordinary temperature:

At ordinary temperatures $f(E)$ remains 1 for $E \ll E_F$, and starts decreasing from 1 as ‘ E ’ becomes closer to E_F as shown in fig.

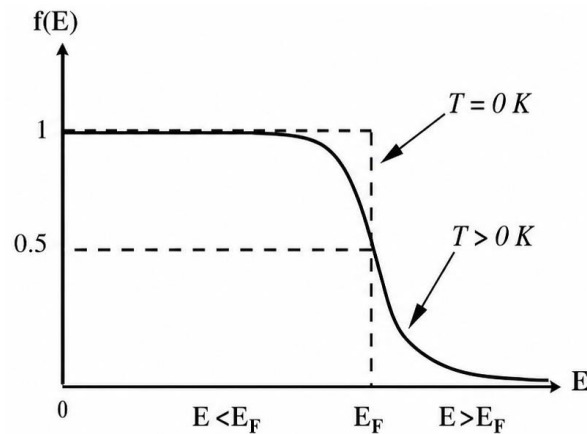
At $E = E_F$ we have,

$$f(E) = \frac{1}{e^0 + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$

$\therefore$ The value of $f(E)$ becomes $\frac{1}{2}$ at $E = E_F$

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Thus, Fermi energy is the most probable energy or the average energy of the electrons which undergo transitions across the Fermi level at temperature above zero degree absolute.



Expression for electrical conductivity and resistivity based on quantum free electron theory:

The momentum of the free electrons in the metal is given by

$$p = \frac{h}{\lambda} ; \quad \text{but } k = \frac{2\pi}{\lambda}$$

Hence, $p = \frac{hk}{2\pi}$

$$mv = \frac{hk}{2\pi}$$

$$v = \frac{hk}{2\pi m} \dots \dots \dots (1)$$

At thermal equilibrium, free electrons move randomly with velocities up to the Fermi velocity (v_F), resulting in zero net current. Under an applied electric field, only electrons near the Fermi level can be accelerated and participate in electrical conduction.

When a electric filed is applied, the electrons are accelerated in the direction of the force. The acceleration of the electrons is given by

Differentiating (1) w.r.t. 't' we get,

$$\frac{dv}{dt} = \frac{h}{2\pi m} \frac{dk}{dt} \Rightarrow \quad m \frac{dv}{dt} = \frac{h}{2\pi} \frac{dk}{dt} \dots \dots \dots (2)$$

we know that Force applied on the electron in the applied field is

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$$F = eE$$

$$ma = eE$$

$$m \frac{dv}{dt} = eE$$

Therefore,

$$\frac{h}{2\pi} \frac{dk}{dt} = eE \Rightarrow dK = \frac{2\pi eE dt}{h} \dots \dots \dots (3)$$

If the field is applied for time period 't', then integrating the above equation from 0 → t, and applying the limits we get

$$k(t) - k(0) = \frac{2\pi eEt}{h}$$

$$\Delta k = \frac{2\pi eEt}{h} \dots \dots \dots (4)$$

Thus, if the field is applied at time $t=0$, electrons acquire a drift velocity in the characteristic relaxation time ,

$$\tau = \tau_F = \frac{\lambda_F}{v_F}$$

$$\Delta k = \frac{2\pi eE}{h} \frac{\lambda_F}{v_F} \dots \dots \dots (5)$$

The steady state current density is given by

$$J = ne\Delta v$$

$$\Delta v = \frac{\hbar \Delta k}{m}$$

$$J = \left[\frac{neh\Delta k}{2\pi m^*} \right] \dots \dots \dots (6)$$

Here m^* is the effective mass of the electrons. The electrical conductivity (σ) is related to current density by

$$J = \sigma E \dots \dots \dots (7)$$

From equation (6) and (7)

$$\sigma = \frac{neh\Delta k}{2\pi m^* E} \dots \dots \dots (8)$$

Substituting from equation (5) in equation (8) we get

$$\sigma = \frac{neh}{2\pi m^* E} \left(\frac{2\pi eE}{h} \frac{\lambda_F}{v_F} \right)$$

$$\sigma = \frac{ne^2}{m^*} \left(\frac{\lambda_F}{v_F} \right)$$

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Also, the expression for resistivity is given by

$$\rho = \frac{m^* v_F}{n e^2 \lambda}$$

SEMICONDUCTORS

Conductivity of solids can be explained on the basis of band theory. The important energy bands in solids are:

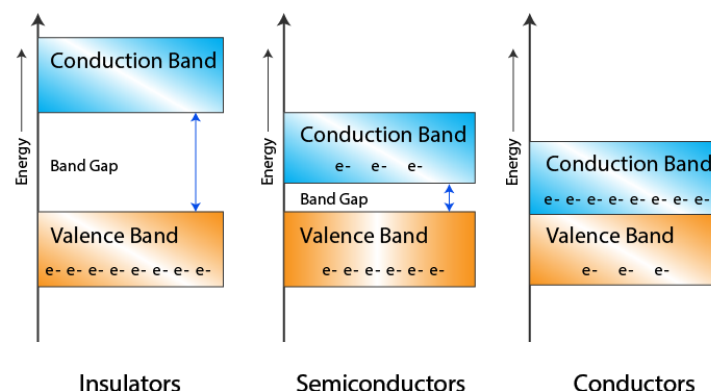
1. **Valence Band:** The electron in the outer most orbit of an atom is known as valence electron. The range of energies possessed by valence electrons is known as valence band. The valence band may be completely or partially filled.

2. **Conduction Band:** The range of energies possessed by conduction electrons is known as conduction band. In some metals, the valence electron may be loosely packed and may get detached to become free electron. These free electrons are responsible for conduction of current in a conductor and are hence called conduction electrons.

Significance of Band Gap Eg:

The band gap energy E_g is minimum amount of energy required for breaking a covalent bond and to excite an electron from valence band to conduction band. The energy required to break a covalent bond in Ge is 0.72eV and for Si it is 1.12eV at 300K.

Forbidden energy band/gap: The separation between bottom of conduction band and top of valence band in the energy level diagram of solids is known as forbidden energy band. There are no electrons in the forbidden energy gap.



Classification of semiconductors:

The materials having moderate electrical conductivity are called as semiconductors. These materials have a completely filled VB and completely filled CB at 0K. Semiconductors

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are classified into two types on the basis of concentrations of electrons and holes in the material.

1. Intrinsic Semiconductor

- A pure semiconductor without any intentional impurity.
- Electrical conduction occurs due to thermally generated electron–hole pairs.
- The number of electrons is equal to the number of holes: $n = p$.

Examples: pure Silicon (Si) and Germanium (Ge).

2. Extrinsic Semiconductor

- A semiconductor whose electrical properties are modified by adding a small amount of impurity (dopant).
- Doping increases the concentration of charge carriers and hence increases electrical conductivity.

Two types:

n-type: Doped with pentavalent impurities (P, As, Sb); electrons are the majority carriers.

p-type: Doped with trivalent impurities (B, Al, Ga); holes are the majority carriers.

Fermi Level in semiconductors:

At 0 K, the conduction band (CB) of a semiconductor is completely empty, while the valence band (VB) is completely filled. As the temperature increases, some electrons gain sufficient thermal energy to move from the VB to the CB, leaving behind holes in the VB. These thermally excited electrons become conduction electrons. The Fermi level is the average energy level associated with these charge carriers. In an intrinsic (pure) semiconductor, the Fermi level lies approximately at the middle of the forbidden energy gap between the conduction band and the valence band.

Electron and hole concentration:

The number of electrons per unit volume in the conduction band of a semiconductor is known as electron concentration. Similarly, the number of holes per unit volume in the valence band of a semiconductor is known as hole concentration.

The electron concentration n_e is given by

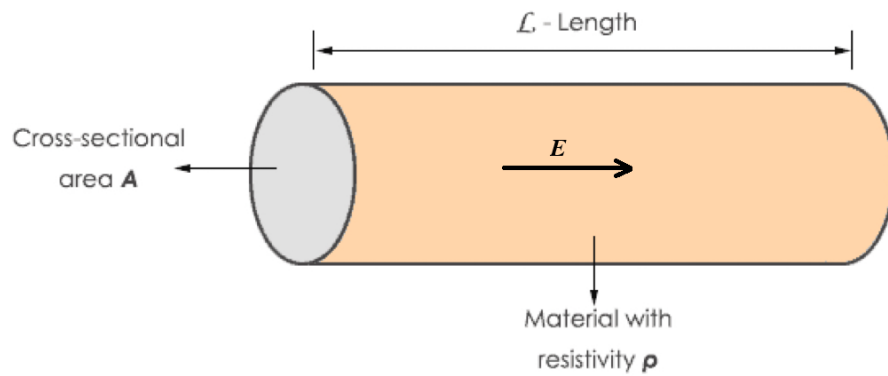
$$n_e = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{\left(\frac{E_F - E_g}{kT}\right)}$$

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The hole concentration n_h is given by

$$n_h = \frac{4\sqrt{2}}{h^3} (\pi m_h^* kT)^{3/2} e^{-\left(\frac{E_F}{kT}\right)}$$

In the above equation E_f is the Fermi energy of the semiconductor, E_g is the energy gap of the semiconductor, m_e^* and m_h^* are the effective masses of electrons and holes, T is the absolute temperature.



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Expression for electrical conductivity of a semiconductor:

Consider a semiconductor of area cross section 'A'. Let an electric field of 'E' is applied to the conductor. Let v_e and v_h be the drift velocities of electrons and holes respectively due to applied field E.

The current due to electron is

$$I_e = n_e e A v_e \rightarrow (1)$$

The current due to holes is

$$I_h = n_h e A v_h \rightarrow (2)$$

Here and n_e & n_h are the number of electrons and holes per unit volume.

The total current is given by

$$I = I_e + I_h = n_e e A v_e + n_h e A v_h$$

The total current is the sum of the current due electrons and holes. The total current density is given by

$$J = n_e e v_e + n_h e v_h \rightarrow (3)$$

By definition, mobility of charge carriers is the drift velocity per unit electric field.

Therefore the drift velocity of electrons is

$$v_e = \mu_e E$$

and the drift velocity of holes is

$$v_h = \mu_h E \quad \because \mu_h = \frac{v_h}{E}$$

Where μ_e & μ_h are the mobilities of electrons and holes respectively.

Substituting for v_e & v_h in equation (3) we get

$$J = n_e e \mu_e E + n_h e \mu_h E \rightarrow (4)$$

From Ohm's law

$$J = \sigma E \rightarrow (5)$$

Where σ is conductivity, which is defined as the current density per unit applied electric field.

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Comparing equation (4) and (5), we get

$$\sigma = n_e e \mu_e + n_h e \mu_h \rightarrow (6)$$

This equation gives total conductivity of a semiconductor.

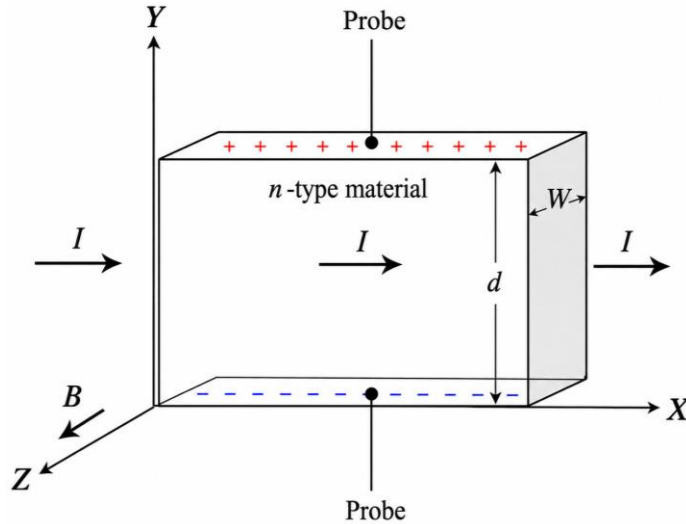
For intrinsic semiconductors,

$n_e = n_h = n_i$; n_i is the intrinsic carrier concentration. Hence

$$\sigma_i = n_i e (\mu_e + \mu_h)$$

Hall Effect

If a current carrying material is placed in a transverse magnetic field, an electric field is produced in the material in a direction perpendicular to both the current and magnetic field. This phenomenon is called Hall Effect. The electric field generated is called Hall field and corresponding voltage is Hall voltage.



Expression for Hall coefficient:

Consider a rectangle slab of a semiconductor material in which a current ' I ' is flowing in positive x-direction. Let the semiconductor material be of n type, which means that the charge carriers are electrons.

Let a magnetic field, B is applied along Z-direction as shown in the figure.

Under the influence of a magnetic field, the electrons experience the Lorentz-force, F_L given by

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$$F_L = -Bev \rightarrow (1); \quad \text{-ve is due to electron charge}$$

Where 'e' is the electron charge and 'v' is the drift velocity of electrons. According to Fleming's left-hand rule, electrons experience the Lorentz force F_L along the negative y-direction, causing charge separation by producing a Hall voltage V_H and Hall electric field E_H .

The field E_H , exerts an upward force F_H on the electrons given by,

$$F_H = -eE_H \rightarrow (2)$$

As electrons continue to deflect downward due to the Lorentz force F_L , the Hall field increases. The upward Hall force F_H increases until it becomes equal to F_L .

Thus, at equilibrium, $F_L = F_H$

$$\begin{aligned} \text{i.e. } -Bev &= -eE_H \quad \text{since } F_L = evB \\ \Rightarrow E_H &= Bv \rightarrow (3) \end{aligned}$$

Using the relation, $I = nAev$,

The current through the semiconductor $I = nAev$

$$v = \frac{I}{Ane}$$

Substituting for v, in equation (3) we get

$$E_H = B \frac{I}{wdne} \text{ ----- (4) since } A = wd$$

$$\text{The Hall voltage, } V_H = E_H d = \frac{BId}{wdne} = \frac{BI}{wne} \text{ ----- (5)}$$

where n is charge carrier concentration.

Hence

$$V_H = \frac{R_H BI}{w}$$

where R_H is a constant called Hall coefficient, $R_H = \frac{1}{\rho_{ch}} = \frac{1}{ne}$

Thus by measuring V_H , I & w and by knowing B , the charge density ρ_{ch} can be determined using which Hall coefficient can be evaluated.

Importance of Hall Effect:

The importance of Hall Effect in the field of semiconductor is that it helps to determine

- 1) The type of semiconductor.

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- 2) The sign of majority charge carriers.
- 3) The majority charge carrier concentration.
- 4) The mobility of majority charge carriers.
- 5) The mean drift velocity of majority charge carriers

DIELECTRICS

A dielectric is an electrically non-conducting material which provides electrical insulation between two media (conductors) which are at different potentials, and also serves as an electrical charge storage aid under certain circumstances.

E.g.: glass, porcelain, wood, rubber or waxed paper.

Electric dipole:

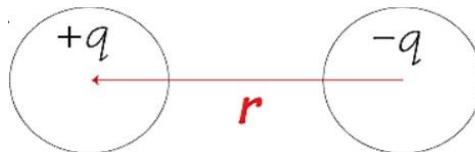
Electric dipole is a pair of equal and opposite charges, $+q$ and $-q$ separated by a very small distance

Examples of electric dipole:- Dipoles are common in nature. Molecules like H_2O , HCl , CH_3COOH are electric dipoles and have permanent dipole moments.

Dipole moment:

A dipole moment is the product of the magnitude of the charge and the distance between the centers of the positive and negative charges in a system. It is denoted by

$$\mu = qr$$



Units: C-m or Debye,

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Polar and non-polar dielectric materials

Polar dielectric	non-polar dielectric
Have permanent dipole moment	Do not have permanent dipole moment
No centre of symmetry	Centre of symmetry
Polarization of molecules is highly dependent on the temperature	Polarization of molecules is Independent to the temperature
Ex: H ₂ O, N ₂ O, HCl	Ex: H ₂ , N ₂ , CH ₄

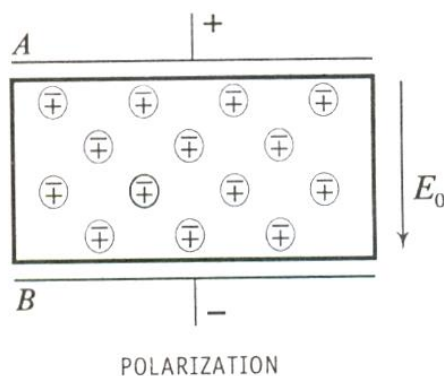
Polarization in dielectrics

The Process of formation of dipoles or alignment of already existing dipoles on the application of an electric field on the dielectric material is called polarization.

(i) Polarization in the case of non-polar dielectric:

Consider a rectangular slab of a non-polar dielectric material. Let the slab be placed between two electrodes A & B, between which a uniform electric field E is set up.

Under the influence of the electric field, the positive and negative charges of the various molecules in it experience pulling forces in opposite directions. As a result, the effective centers of positive and negative charges get separated till the restoring forces balance the forces due to the applied field. Due to the separation between the two charge centers, each molecule develops a dipole moment in the direction of the electric field. Such dipoles are produced throughout the solid.



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Now, inside the body of the dielectric slab, though the polarization charges appear in every molecule, opposite charges in the neighboring molecules neutralize each other. This kind of cancellation takes place throughout the body of the material since there are equal amount of negative and positive charges. However, the charges which are at the end surfaces of the slab do not find the opposite kind of charges for cancellation as is evident from the figure. Thus the effect of the applied field is to cause the appearance of net opposing charges at the end faces of the slab.

(ii) Polarization in the case of polar dielectric:

In the case of a polar dielectric material, there are permanent dipoles oriented randomly. Under the influence of the applied electric field, the molecular dipoles experience a torque which tends to align their dipole moments along the direction of the field. This tendency is opposed by the thermal agitation inside the material. Between these two opposing events, some alignment is achieved under sufficiently strong fields. Once the alignment is established, the surface charges appear at the ends of the slab in a way similar to the case of non-polar dielectric materials.

Types of polarization processes

There are three different mechanisms through which electrical polarization can occur in dielectric materials when they are subjected to an external electric field. Accordingly there are three main types of polarizations.

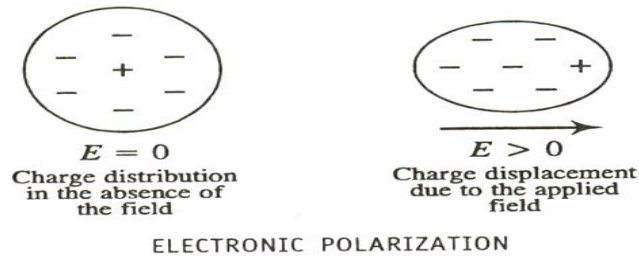
They are,

- 1) Electronic polarization,
- 2) Ionic polarization
- 3) Orientational polarization

1. Electronic polarization:

The electronic polarization occurs due to displacement of the positive and negative charges in a dielectric material owing to the application of an external electric field. The separation created between the charges leads to development of a dipole moment. This process occurs throughout the material. Thus the material as a whole will be polarized.

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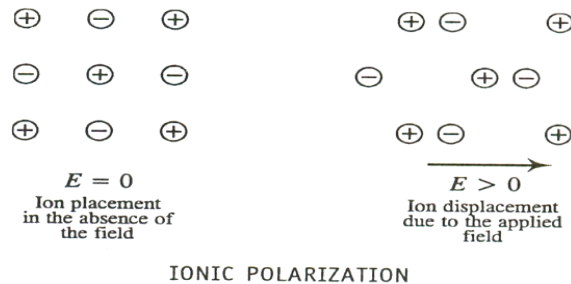


The electronic polarizability, α_e for a rare gas atom is given by,

$$\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}; \text{ where } N \text{ is the number of atoms/unit volume.}$$

2. Ionic Polarization:

Ionic polarization occurs only in those dielectric materials which possess ionic bonds such as in NaCl. When ionic solids are subjected to an external electric field, the adjacent ions of opposite sign undergo displacement. The displacement causes an increase or decrease in the distance of separation between the atoms depending upon the location of the ion pair in the lattice.

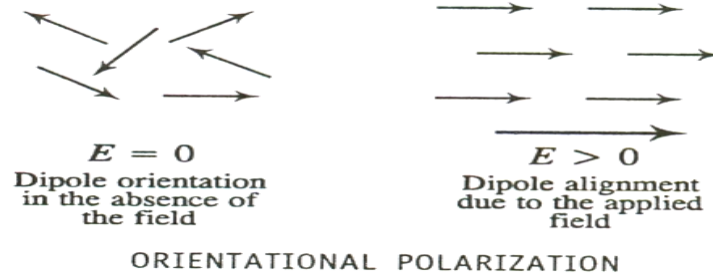


This results in a net dipole moment due to the shifting of electron clouds relative to the nuclei.

3. Orientational Polarization:

Orientational polarization occurs in those dielectric materials which possess molecules with permanent dipole moment (i.e., in polar dielectrics). The orientation of these molecules will be random normally due to thermal agitation. Because of randomness in orientation, the material has net zero dipole moment. But under the influence of an applied electric field, each of the dipoles undergo rotation so as to reorient along the direction of the field because of which, the overall cancellation of dipole moments due to randomness does not hold good any more (Fig. 7). Thus the material itself develops electrical polarization.

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The orientational polarization is strongly temperature dependent and decreases with increase of temperature.

Internal field:

What is internal field? Write an expression for the internal field in case of solid and liquid dielectrics?

"The internal field, or the local field, is the electric field that acts at the site of any given atom of a solid or a liquid dielectric subjected to an external electric field. It is defined as "the resultant of the applied field and the field due to all the surrounding dipoles".

The general equation for internal field is expressed as,

$$E_i = E + \frac{\gamma P}{\epsilon_0}$$

where, P is the dipole moment/unit volume for the material, and γ is the proportionality constant called internal field constant.

In the 3-dimensional case, for a cubic lattice, it can be shown that $\gamma = 1/3$. Under this condition, the internal field is called the Lorentz field and is given by:

$$E_{Lorentz} = E + \frac{P}{3\epsilon_0}$$

Above equation is known as Lorentz relation.

Clausius – Mossotti relation

Consider an elemental solid dielectric material of dielectric constant ϵ_r

If N is the number of atoms/unit volume of the material, μ is the atomic dipole moment,

Then, dipole moment/unit volume = $N\mu$ ----- (1)

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Here the field experienced by the atoms is the internal field E_i . Hence, if α_e is the electronic polarizability of the atoms, we can write the equation for μ as,

$$\mu = \alpha_e E_i \text{ ----- (2)}$$

Therefore dipole moment/unit volume = $N\alpha_e E_i$ ----- (3)

But dipole moment/ unit volume is Polarization P

Therefore $P = N\alpha_e E_i$ ----- (4)

Or $E_i = \frac{P}{N\alpha_e}$ ----- (5)

But $P = \epsilon_0(\epsilon_r - 1)E$

Or $E = \frac{P}{\epsilon_0(\epsilon_r - 1)}$ ----- (6)

For 3-D lattice, internal field is given by

$$E_i = E + \frac{\gamma P}{\epsilon_0}$$

Substituting for E_i and E from Eqs (5) and (6) we have

$$\frac{P}{N\alpha_e} = \frac{P}{\epsilon_0(\epsilon_r - 1)} + \frac{\gamma P}{\epsilon_0} \text{ ----- (7)}$$

Considering the internal field in the material to be Lorentz field, we have $\gamma = 1/3$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_0(\epsilon_r - 1)} + \frac{1}{3\epsilon_0}$$

$$\frac{1}{N\alpha_e} = \frac{3 + (\epsilon_r - 1)}{3\epsilon_0(\epsilon_r - 1)}$$

$$\frac{3\epsilon_0}{N\alpha_e} = \frac{\epsilon_r + 2}{\epsilon_r - 1}$$

By rearranging we have, $\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$

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Numerical Problems:

1. Calculate the probability of an electron occupying an energy level 0.04 eV above the Fermi level at 400 K in a material.

Solution:

Given, $T = 400K$,

Above the Fermi Level, $E - E_F = 0.04eV$

To find $f(E) = ?$

We have

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$
$$f(E) = \frac{1}{1 + e^{\frac{0.04 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 400}}}$$
$$= 0.238$$

2. Calculate the Fermi velocity and mean free path for conduction electrons in Al, given that its Fermi energy is 11.63 eV and relaxation time for free electrons is $7.3 \times 10^{-15}s$.

Solution:

Given: $E_F = 11.63 eV$, $\tau = 7.3 \times 10^{-15}s$

To find; $v_F = ?$ $\lambda_F = ?$

We have $E_F = \frac{mv_F^2}{2}$

$$v_F = \sqrt{\frac{2E_F}{m}}$$
$$v_F = \sqrt{\frac{2 \times 11.63 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$
$$= 2.022 \times 10^6 m/s$$

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3. Find the temperature at which there is 1% probability that a state with energy 0.5 eV above Fermi energy is occupied.

Solution:

$$\text{Given: } f(E) = 1\% = \frac{1}{100}$$

$$\text{Above the Fermi Level, } E - E_F = 0.5 \text{ eV}$$

To find, T =?

We have

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

$$\frac{1}{100} = \frac{1}{1 + e^{\frac{0.5 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times T}}}$$

$$T = 1261 \text{ K}$$

4. The intrinsic carrier density is $1.5 \times 10^{16} \text{ m}^{-3}$. If the mobility of electron and hole are 0.13 and $0.05 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, calculate the conductivity.

Solution:

Given:

$$n_i = 1.5 \times 10^{16} \text{ m}^{-3}$$

$$\mu_e = 0.13 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_h = 0.05 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\text{Conductivity } \sigma = n_i e (\mu_e + \mu_h)$$

$$\sigma = 1.5 \times 10^{16} \times 1.6 \times 10^{-19} (0.13 + 0.05)$$

$$\sigma = 4.32 \times 10^{-4} \Omega^{-1} \text{ m}^{-1}$$

5. The Intrinsic carrier density at room temperature in Ge is $2.37 \times 10^{19} \text{ m}^{-3}$ if the electron and hole mobilities are 0.38 and $0.18 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ respectively, calculate the resistivity.

Solution:

Given:

$$n_i = 2.37 \times 10^{19} \text{ m}^{-3}$$

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$$\mu_e = 0.38 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\mu_h = 0.18 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\text{Conductivity } \sigma = n_i e (\mu_e + \mu_h)$$

$$\sigma = 2.37 \times 10^{19} \times 1.6 \times 10^{-19} (0.38 + 0.18)$$

$$\sigma = 2.1235 \Omega^{-1} \text{ m}^{-1}$$

Resistivity

$$\rho = \frac{1}{\sigma}$$

$$\rho = \frac{1}{2.1235}$$

$$\rho = 0.4709 \Omega \text{ m}$$

6. The Hall coefficient of certain silicon specimen was found to be $-7.35 \times 10^{-5} \text{ m}^3 \text{ C}^{-1}$ from 100 to 400 K. Determine the nature of the semiconductor. If the conductivity was found to be $200 \Omega^{-1} \text{ m}^{-1}$. Calculate the density and mobility of the charge carrier.

Solution:

$$\text{Conductivity } \sigma = 200 \Omega^{-1} \text{ m}^{-1}$$

$$\text{Hall co-efficient } R_H = -7.35 \times 10^{-5} \text{ m}^3 \text{ C}^{-1}$$

a) Density of electrons

$$n = \frac{-1}{R_H e} \text{ (from equation (1))}$$

$$n = \frac{1}{(7.35 \times 10^{-5} \times 1.609 \times 10^{-19})}$$

$$\text{(i.e.)} = 8.455 \times 10^{22} \text{ m}^{-3}$$

We know Conductivity

$$\sigma = n e \mu_e$$

b) Mobility

$$\begin{aligned} \mu &= \frac{\sigma}{n e} = \frac{200}{8.455 \times 10^{22} \times 1.6 \times 10^{-19}} \\ &= 0.0147 \end{aligned}$$

$$\text{Mobility } \mu = 0.0147 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

$$\text{Density of electrons (n)} = 8.053 \times 10^{22} \text{ m}^{-3}$$

$$\text{Mobility } (\mu) = 0.0147 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

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7. A semiconducting crystal with 12 mm long, 5 mm wide and 1 mm thick has a magnetic density of 0.5 Wbm^{-2} applied from front to back perpendicular to largest faces. When a current of 20 mA flows length wise through the specimen, the voltage measured across its width is found to be $37\mu\text{V}$. What is the Hall coefficient of this semiconductor?

Solution:

Given:

$$\text{Hall voltage } V_H = 37 \mu\text{V} = 37 \times 10^{-6} \text{ V}$$

$$\text{Breath of the material } t = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$\text{Current } I_H = 20 \text{ mA} = 20 \times 10^{-3} \text{ A}$$

Magnetic flow density

$$B = 0.5 \text{ Wbm}^{-2}$$

$$\text{Hall coefficient } R_H = \frac{V_H t}{I_H B}$$

$$= \frac{37 \times 10^{-6} \times 1 \times 10^{-3}}{20 \times 10^{-3} \times 0.5}$$

$$R_H = 3.7 \times 10^{-6} \text{ C}^{-1} \text{ m}^3$$

$$\therefore \text{Hall coefficient } R_H = 3.7 \times 10^{-6} \text{ C}^{-1} \text{ m}^3$$

8. The Electronic polarizability of Krypton gas is $3.54 \times 10^{-40} \text{ Fm}^2$. If the gas contains $2.7 \times \frac{10^{25} \text{ atoms}}{\text{m}^3}$ at NTP calculate the dielectric constant of Krypton.

Solution:

Given:

$$\text{Electronic polarizability } \alpha_e = 3.54 \times 10^{-40} \text{ Fm}^2$$

$$\text{Number of atoms per unit volume, } N = 2.7 \times 10^{25} \text{ atoms/m}^3$$

$$\text{Dielectric constant, } \epsilon_r = K = ?$$

We have

$$\alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

Substituting the given values,

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$$3.54 \times 10^{-40} = \frac{8.854 \times 10^{-12}(\epsilon_r - 1)}{2.7 \times 10^{25}}$$

Simplifying, we get

$$\epsilon_r = 1.00108$$

9. An elemental solid dielectric material has polarizability $7 \times 10^{-40} \text{ Fm}^2$. Assuming the internal field to be Lorentz field, calculate the dielectric constant for the material if the material has $3 \times 10^{28} \text{ atoms/m}^3$.

Solution:

Electronic polarizability $\alpha_e = 7 \times 10^{-40} \text{ Fm}^2$

Number of atoms per unit volume, $N = 3 \times 10^{28} \text{ atoms/m}^3$

Dielectric constant, $\epsilon_r = ?$

Since, the internal field is Lorentz field we can apply Clausius - Mossotti equation given by

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\frac{3 \times 10^{28} \times 7 \times 10^{-40}}{3 \times 8.854 \times 10^{-12}} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\epsilon_r = 12.33$$

10. Find the polarization produced in a dielectric medium of relative permittivity 15 in the presence of an electric field of 500 V/m.

Solution:

Given: Relative permittivity = Dielectric constant, $\epsilon_r = 15$

Electric field, $E = 500 \text{ V/m}$

Polarization, $P = ?$

We have,

$$P = \epsilon_0(\epsilon_r - 1)E$$

Substituting we get,

$$P = 8.854 \times 10^{-12} \times (15 - 1)500$$

$$P = 6.1978 \times 10^{-8} \text{ C/m}^{-2}$$

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Sample Questions:

1. Explain the behaviour of Fermi-Dirac distribution function for different energy levels and temperatures.
2. Apply quantum-free electron theory to obtain an expression for the electrical conductivity of metals.
3. Applying the concept of charge carriers in semiconductors, derive an expression for electrical conductivity in intrinsic semiconductors.
4. Analyze the Hall Effect in a semiconductor and hence obtain the expressions for Hall coefficient and Hall voltage.
5. Define Polarization in dielectric materials. Explain polarization in dielectrics by electronic and orientational polarization methods.
6. Using the local field approach, establish the Claussius-Mossotti relation for dielectric materials.
