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NAAC A+ GRADE CGPA: 3.44

MANGALORE INSTITUTE OF TECHNOLOGY & ENGINEERING

(A Unit of Rajalaxmi Education Trust ®, Mangalore)

Autonomous Institute affiliated to VTU, Belgavi, Approved by AICTE, New Delhi

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MODULE 1 : MATRIX ALGEBRA

- **Contents**

- Solution to system of linear equations.
- Elementary row transformation of a matrix
- Rank of a matrix. Gauss-Elimination method.
- Approximate solution by Gauss-Seidel method.
- Solution of system of Ordinary Differential equations by Matrix method.

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Learning Objective

- Impart the knowledge of various matrix methods and techniques for solving system of linear equations & ordinary differential equations.

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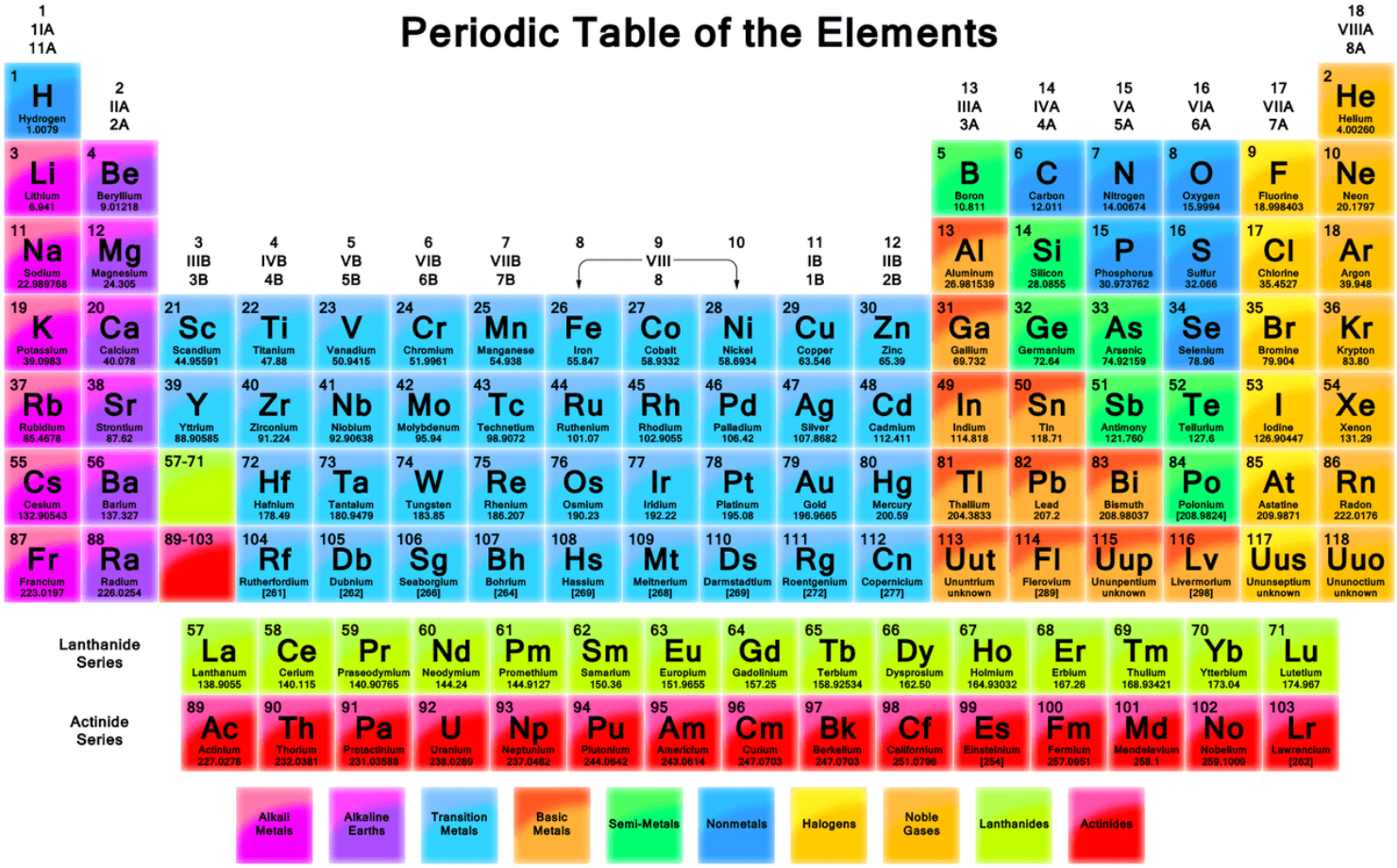
Observe the following images and tell me what you can infer from this





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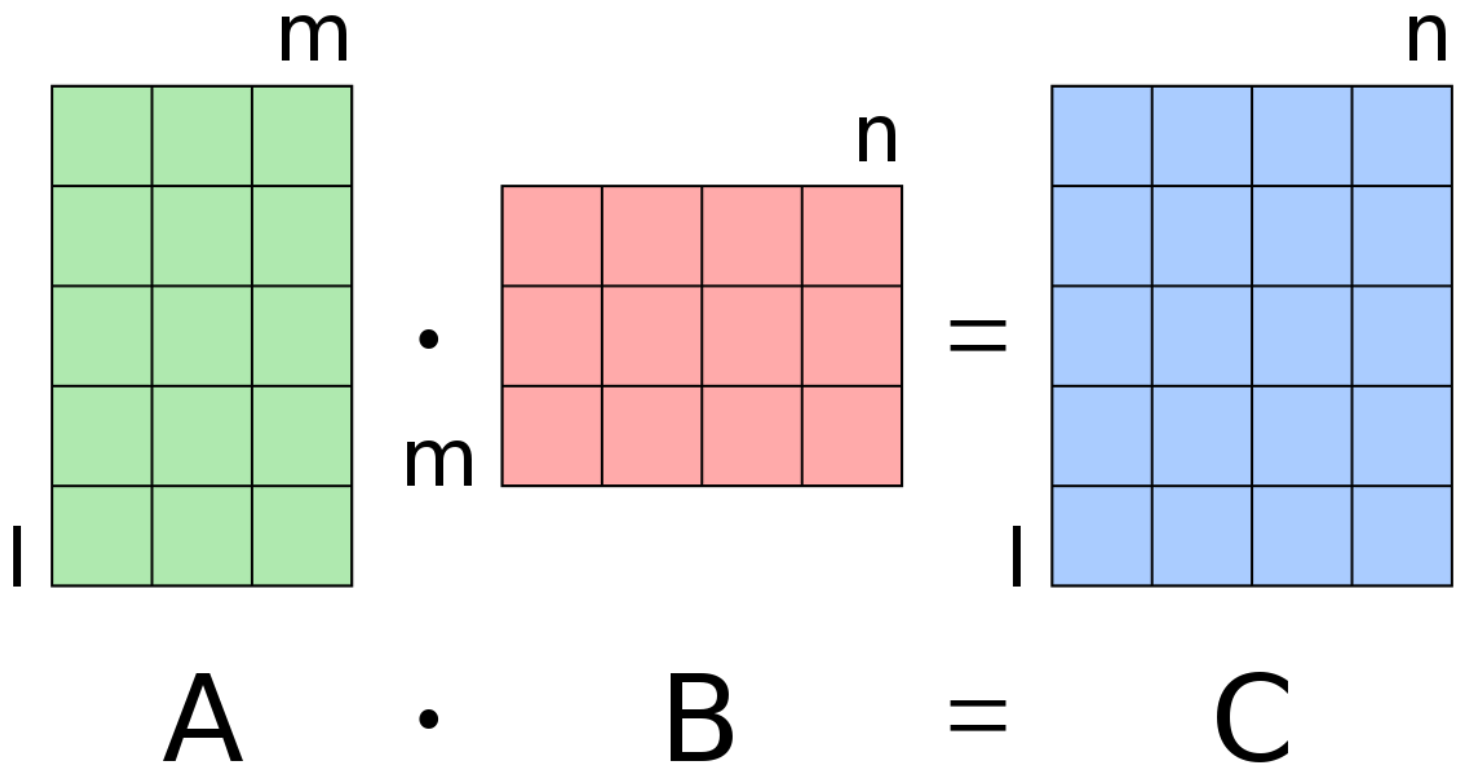


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Marks list of a students

	A	B	C	D
1	Student	Exam 1	Exam 2	Exam 3
2	Jim	70	72	80
3	Sue	78	85	99
4	Andy	89	92	88
5	Bo	90	88	86
6	Aki	81	67	78
7				


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A · B = C

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INTRODUCTION

- A matrix is a fundamental mathematical concept used to organize and manipulate data in various fields, including mathematics, physics, computer science, and engineering. It consists of rows and columns of numbers, symbols, or variables arranged in a rectangular grid.
- Matrices are integral to many aspects of modern science and technology, making them an essential concept for anyone working with data or mathematical modelling.

DEFINITION

- A matrix is a rectangular arrangement of numbers in rows and columns represented by

$$\begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}.$$

- If a matrix has m rows and n columns, then it is said to be of order $m \times n$ (read as "m by n" matrix). The elements a_{ij} of a matrix are identified by double subscript notation ij, where i denotes the row and j denotes the column.

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SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

Suppose Kumari had rupees 12100 to invest. She decided to invest her money in bonds and mutual funds. She invested a portion of the money in bonds paying 8% interest per year and the remainder in a mutual fund paying 9% per year. After one year the total income she had earned from the investments was rupees 1043. How much had she invested at each rate?

Let the amount invested in bonds be represented by x and the amount invested in mutual funds be represented by y. Then we can express the problem in the form of equations as follows:

The total amount of money she had to invest was

$$x + y = 12100.$$

The amount of money she earned from the investments was

$$\frac{8}{100}\,x + \frac{9}{100}\,y = 1043.$$

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SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

A power source forces a current to flow through the network, where it encounters various resistors, each which requires a certain amount of force to be applied in order for the current to flow through. Systems of linear equations are used to determine the currents through various branches of electrical networks.

SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

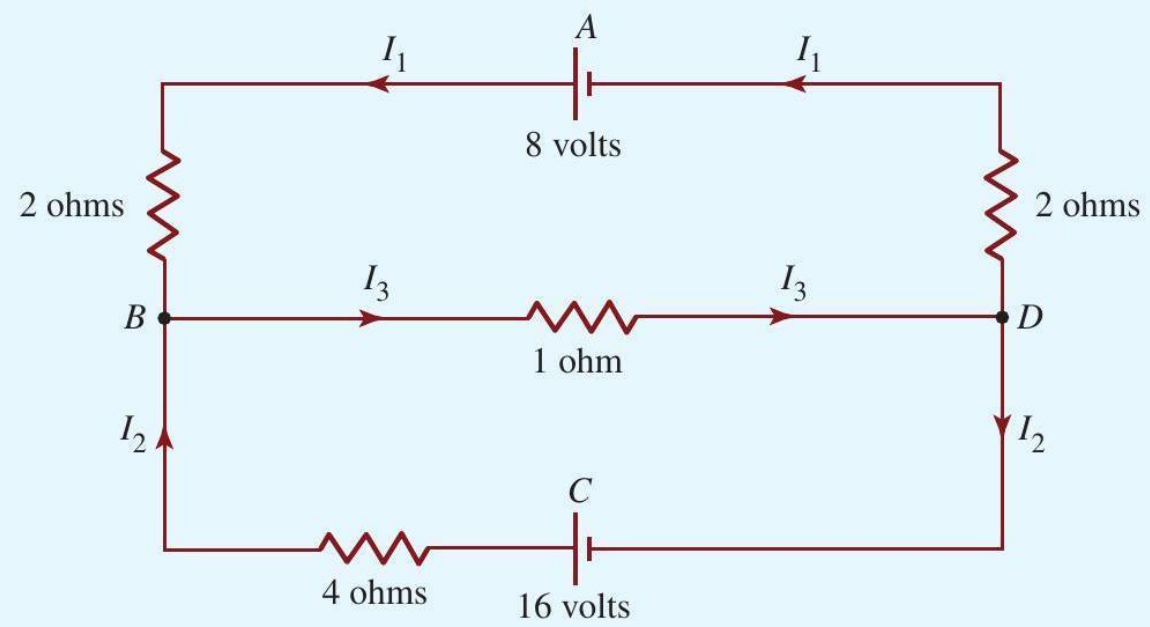
Using Ohm's law and Kirchhoff's law,

Junction: All the current flowing into a junction must flow out of it.

Path: The sum of the IR terms (I denotes current, R resistance) in any direction around a closed path is equal to the total voltage in the path in that direction.

SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

We wish to determine the currents I_1, I_2 *and* I_3 in the below circuit. Applying Ohm's and Kirchhoff's Law, we can construct a system of linear equations.



SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

There are two junctions in this circuit, namely the points B and D. There are three closed paths, namely $ABDA, CBDC$, and $ABCDA$. Apply the laws to the junctions and paths.

$$\text{Junction } B, \qquad I_1 + I_2 = I_3$$
$$\text{Junction } D, \qquad I_3 = I_1 + I_2$$

These two equations result in a single linear equation

$$I_1 + I_2 - I_3 = 0.$$

Paths:

$$\text{Path } ABDA, \quad 2I_1 + 1I_3 + 2I_1 = 8$$
$$\text{Path } CBDC, \quad 4I_2 + 1I_3 = 16$$

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SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

The problem thus reduces to solving the following system of three linear equations in three variables.

$$
\begin{aligned}
I_1 + I_2 - I_3 &= 0 \\
4I_1 + I_3 &= 8 \\
4I_2 + I_3 &= 16
\end{aligned}
$$

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SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

In general, we can construct the system a linear system of simultaneous equations of **m** equations in **n** unknowns can be expressed as

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n &= b_2, \\ a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots\ldots + a_{mn}x_n &= b_m. \end{aligned}\right\} \quad (1)$$

FORMATION OF SYSTEM OF SIMULATANEOUS LINEAR EQUATIONS

1. A person invested ₹20,000, part at 10% simple interest and the rest at 12%. After one year, the total interest earned was ₹2,200. Form a system of simultaneous linear equations for this scenario.

   Ans: $\boldsymbol{x + y = 20000,\ \ 0.10x + 0.12y = 2200}$

2. A shopkeeper sold 12 items consisting of pencils and pens. Each pencil costs ₹5 and each pen costs ₹10. The total sales amount was ₹90. Form system of simultaneous linear equations:

   Ans: $\boldsymbol{x + y = 12,}$
   $\boldsymbol{5x + 10y = 90}$

3. A shopkeeper has a 40% sugar solution and a 60% sugar solution. He mixes them to obtain 30 liters of a 50% solution. Form a system of simultaneous linear equations for this case?

Ans: $x + y = 30$

$0.4x + 0.6y = 0.5 \times 30 = 15$ **(total sugar in liters).**

SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

The above system in the matrix equivalent form can be expressed as $\boldsymbol{AX} = \boldsymbol{B}$, where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & \cdot & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdot & \cdot & \cdot & a_{mn} \end{bmatrix}_{m \times n}$$

is called the coefficient matrix,

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$$ is called the matrix of unknowns,

SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS

and $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}$ is column matrix of constants.

If all b_i's for $i = 1,2,\ldots.m$ are zero,

i.e., $b_1 = b_2 = \cdots = b_m = 0$,

then the system is said to be homogenous and is said to be non-homogeneous if at least one b_i is non-zero.

AUGMENTED MATRIX

Suppose we form a matrix of the form [A : B] by appending to A an extra column whose elements are columns of B i.e.,

$$[\mathrm{A} : \mathrm{B}] = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} : b_1 \\ a_{21} & a_{22} & . & . & . & a_{2n} : b_2 \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} : b_m \end{bmatrix}$$

is called the augmented matrix associated with the system and is denoted by $[A|B]$ or [A : B].

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ELEMENTARY ROW TRANSFORMATIONS

These are operations that are carried out on the rows of a given matrix. The following operations constitute the three row transformations.

- Interchange of i^{th} and j^{th} rows: $R_i \leftrightarrow R_j$.

- Multiplying each element of the i^{th} row by a non-zero constant k: $R_i' \rightarrow kR_i$

- Adding a constant k multiple of j^{th} row to i^{th} row:

$R_i' \rightarrow R_i + kR_j$.

NOTE: All the above operations can also be performed on columns and these transformations does not alter the nature of matrix.

ECHELON FORM OR ROW ECHELON FORM

A non-zero matrix A is said to be in echelon form, if the leading entry (non-zero element) of each non-zero row after the first row occurs to the right of the leading entry of the previous row.

- All the entries of a column below a leading entry are zero.
- All zero rows (all elements are zero) are at the bottom of the matrix.
- A non-zero matrix A is an echelon matrix, if the number of zeros preceding the first non-zero entry of a row increases row by row until zero rows remaining.

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RANK OF A MATRIX

- Rank of a matrix is the number of non-zero rows in the row Echelon Form and is denoted by ρ.

- **Example:**

$$B = \begin{bmatrix} 1 & 3 & 1 & 5 & 0 \\ 0 & 1 & 5 & 1 & 5 \\ 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ is in row-echelon form.

The number of non-zero rows is 3. i.e., $\rho(B) = 3$.

1. Define the rank of the matrix
2. Define row echelon form of a matrix
3. Find the rank of the given echelon matrix: $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
4. Does a homogeneous system of linear equations always have a trivial solution?
5. Identify whether the following matrix are in row echelon form. $(i)\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 & 5 \\ 3 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ determine its rank.
6. Express the given set of simultaneous linear equations in augmented matrix form:

- $2x_1 - x_2 + 3x_3 = 1;\ -3x_1 + 4x_2 - 5x_3 = 0;\quad x_1 + 3x_2 - 6x_3 = 0.$
- $x + 2y + 2z = 1;\quad 2x + y + z = 2;\quad 3x + 2y + 2z = 3.$

7. Determine whether the given matrix $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}$ is in row echelon form by stating three conditions that a matrix must satisfy to be in row echelon form

Ans: Yes.

i) The leading entry of any row must be at right side of leading entry of previous row.

ii) The column entries below the leading entry must be zero.

iii) All the zero rows must be at the bottom of the row reduced echelon form of the matrix.

Worked Example

1. Determine the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

by reducing it to row echelon form.

Ans: $A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) =$ Rank of $A =$ number of non-zero rows$= 3$.

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Worked Example

2. Using the elementary transformations find the rank of the matrix

$$B = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

Solution:

$$B \sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$ The rank of matrix $B = 2$.

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Practice Problems

Find the rank of the following matrices by reducing it to echelon form:

1. $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

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Practice Problems

Answers:

1. 2
2. 2
3. 3
4. 2
5. 4
6. 2

QUIZ QUESTIONS

1. A matrix is defined as:

a) A single row of numbers

b) A rectangular array of numbers arranged in rows and columns

c) A sequence of equations

d) A set of vectors

Ans: b

2. Systems of linear equations are used in electrical networks to determine:

a) Voltages only

b) Currents through various branches

c) Resistance values

d) Power ratings

Ans: b

3. The rank of a matrix represents:

a) The trace of the matrix

b) The number of linearly independent rows or columns

c) The determinant value

d) The size of the matrix

Ans: b

4. Which of the following matrices is in row echelon form?

a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 5 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

Ans: b

c) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

5. The augmented matrix is obtained by:

a) Appending the constants column to the coefficient matrix

b) Adding all rows of the coefficient matrix

c) Multiplying coefficient matrix with solution vector

d) Taking transpose of coefficient matrix

Ans: a

6. What is the rank of the given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{bmatrix}$

a. 2

b. 3

c. 1

d. none of these

Ans: 2

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Home Work Problems

Find the rank of the following matrices by reducing it to echelon form:

7. $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

8. $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$

9. $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$

10. $\begin{bmatrix} 21 & 22 & 23 & 24 \\ 22 & 23 & 24 & 25 \\ 23 & 24 & 25 & 26 \\ 24 & 25 & 26 & 27 \end{bmatrix}$

Home Work Problems

Answers:

7. 3

8. 2

9. 2

10. 2

APPLICATIONS

- One useful application of calculating the rank of a matrix is the computation of the number of solutions of a system of linear equations.

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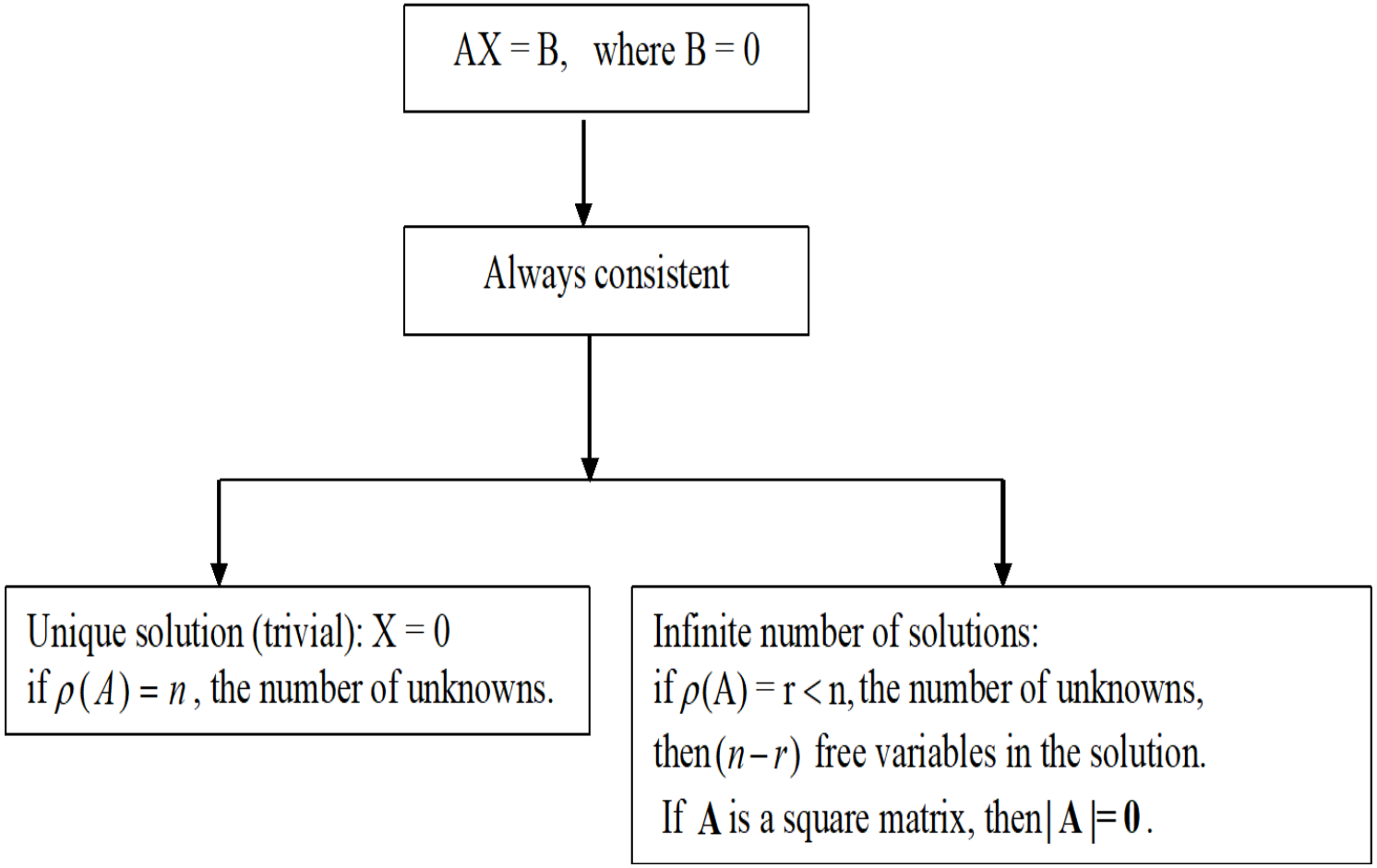
- If we view a matrix as a matrix of linear transformation, the rank talks about the dimension of the range.
- In the field of communication complexity, the rank of the communication matrix of a function gives bounds on the amount of communication needed for two parties to compute the function.

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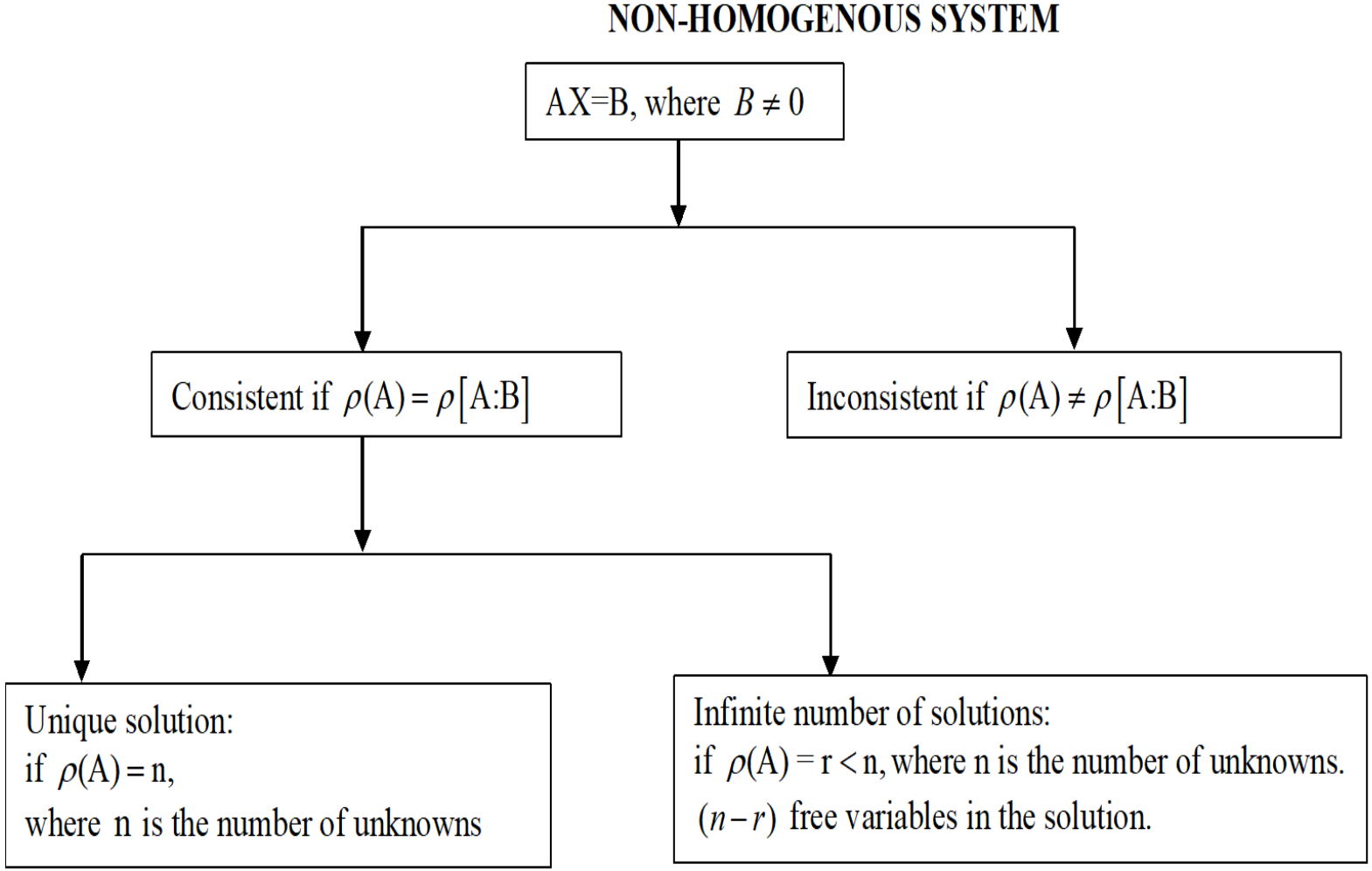
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SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS

HOMOGENOUS SYSTEM



SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS



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GRAPHICAL REPRESENTATION

CLASSIFYING SYSTEMS:

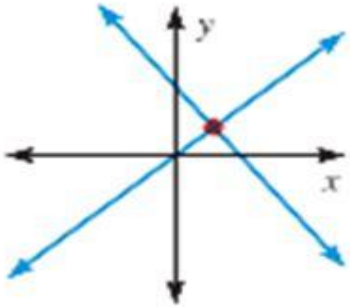
CONSISTENT

one or more solutions

one solution

INDEPENDENT

Exactly one solution

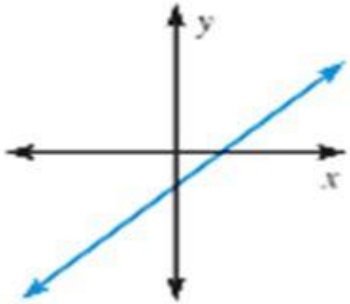


Lines intersect at one point; consistent and independent

more than 1 solution

DEPENDENT

Infinitely many solutions

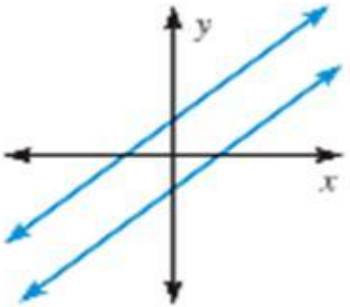


Lines coincide; consistent and dependent

INCONSISTENT

no solutions

No solution



Lines are parallel; inconsistent

WORKED EXAMPLES

1. Test for consistency and solve

$$2x_1 - x_2 + 3x_3 = 1; \quad -3x_1 + 4x_2 - 5x_3 = 0; \quad x_1 + 3x_2 - 6x_3 = 0$$

Solution: Consider the augmented matrix

$$[\text{A:B}] = \begin{bmatrix} 2 & -1 & 3 & :1 \\ -3 & 4 & -5 & :0 \\ 1 & 3 & -6 & :0 \end{bmatrix}$$

Then

$$[\text{A:B}] \sim \begin{bmatrix} 2 & -1 & 3 & : & 1 \\ 0 & 5 & -1 & : & 3 \\ 0 & 0 & -68 & : & -26 \end{bmatrix}$$

Hence, $\rho(A) = \rho([\mathrm{A} : \mathrm{B}]) = 3$ = number of unknowns. The equations can be rewritten as

$2x_1 - x_2 + 3x_3 = 1; \qquad 5x_2 - x_3 = 3; \qquad -68x_3 = -26.$

Here we make use of **back substitution** in order to find the unknowns by considering, last equation to find x_3, next second to find x_2 and finally first equation to find x_1. From last equation we obtain x_3:

$$\text{i.e., } -68x_3 = -26 \Rightarrow x_3 = \frac{13}{34}.$$

Next, from second equation with substitution of x_3, we find x_2:

$$\text{i.e., } 5x_2 - x_3 = 3 \Rightarrow x_2 = \frac{3+x_3}{5} = \frac{3+\frac{13}{34}}{5} \Rightarrow x_2 = \frac{23}{34}.$$

Finally, to find the x_1 we make use first equation

$$2x_1 - x_2 + 3x_3 = 1 \Rightarrow x_1 = \frac{1}{2}(1 + x_2 - 3x_3) = \frac{1}{2}\left(1 + \frac{23}{34} - 3\frac{13}{34}\right)$$

$$\Rightarrow x_1 = \frac{9}{34}.$$

Thus, the unique solution is given by

$$x_1 = \frac{9}{34}, \quad x_2 = \frac{23}{34}, \quad x_3 = \frac{13}{34}.$$

WORKED EXAMPLES

2. Check the following system of equations for consistency and solve, if consistent.

$$x + 2y + 2z = 1, \qquad 2x + y + z = 2,$$

$$3x + 2y + 2z = 3,\ y + z = 0.$$

Solution : The augmented matrix is $[\text{A:B}] = \begin{bmatrix} 1 & 2 & 2 & :1 \\ 2 & 1 & 1 & :2 \\ 3 & 2 & 2 & :3 \\ 0 & 1 & 1 & :0 \end{bmatrix}$

$[\text{A:B}] \sim \begin{bmatrix} 1 & 2 & 2 & :1 \\ 0 & 1 & 1 & :0 \\ 0 & 0 & 0 & :0 \\ 0 & 0 & 0 & :0 \end{bmatrix}$. $\rho(A) = \rho([\text{A:B}]) = 2 < 3$ number of unknowns.

WORKED EXAMPLES

Thus, the given system is consistent and possesses infinite number of solutions by assigning arbitrary values to $(n-r) = 3-2 = 1$, free variable.

$$\Rightarrow \quad x + 2y + 2z = 0$$

$$y + z = 0.$$

Here there are three unknowns, we should take z as the free variable and let $z = k$ (arbitrarily value). From second equation, $y + z = 0 \Rightarrow y = -z = -k$.

Finally, from first equation, $x + 2y + 2z = 1 \Rightarrow x = 1 - 2y - 2z = 1 - 2(-k) - 2k \Rightarrow x = 1$.

Therefore, the solutions are given by $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -k \\ k \end{bmatrix}$.

WORKED EXAMPLES

3. Show that the following system of equations is not consistent.

$$x + 2y + 3z = 6, \quad 3x - y + z = 4, 2x + 2y - z = -3$$

$$, \quad -x + y + 2z = 5$$

Solution: Consider the augmented matrix

$$[\text{A:B}] = \begin{bmatrix} 1 & 2 & 3 & :6 \\ 3 & -1 & 1 & :4 \\ 2 & 2 & -1 & :-3 \\ -1 & 1 & 2 & :5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & :6 \\ 0 & -7 & -8 & :-14 \\ 0 & 0 & -33 & :-77 \\ 0 & 0 & 0 & :28 \end{bmatrix}$$

$$\rho(A) = 3 \text{ and } \rho([\text{A:}B]) = 4$$

$$\rho(A) \neq \rho([\text{A:B}]).$$

Therefore, the given system is inconsistent and it has no solution.

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Practice Problems

1. Show that the system $x + y + z = 4;\ 2x + y - z = 1;$
$x - 2y + 2z = 2$ is consistent and solve the system.

2. Test for consistency and solve:

a) $5x + 3y + 7z = 4,\ \ 3x + 26y + 2z = 9,\ \ 7x + 2y + 10z = 5.$

b) $5x + y + 3z = 20,\ \ 2x + 5y + 2z = 18,\ \ 3x + 2y + z = 14.$

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Practice Problems

Answers:

1. $x = \frac{4}{5}, y = \frac{13}{10}, z = \frac{19}{10}.$

2. $x = \frac{\lambda^2}{6} - \frac{\lambda}{2} + \frac{1}{3}, y = -\frac{5k^2}{12} + \frac{3k}{4} + \frac{7}{6}, z = \frac{k^2}{4} - \frac{k}{4} - \frac{1}{2}.$

3. a) $x = \frac{7}{11}, y = \frac{3}{11}, z = 0.$

b) $x = 3, y = 2, z = 1.$

1. For what value of λ the system of linear equations: $2x + 3y + 4z = 5$,

$5y + 6z = 6\ and\ \ (\lambda - 7)z = 0$, has infinite solutions, unique solution.

2. Determine the value of $\lambda\ and\ \mu$, for which the given matrix $\left[\begin{array}{cccc} 2 & 3 & 4 & : & 6 \\ 0 & 5 & 6 & : & 4 \\ 0 & 0 & \lambda - 3 & : & \mu - 10 \end{array}\right]$ of system of linear equation has unique solution, Infinite solution, No solution.

Ans: Unique Solution: When $\lambda \neq 3$ and $\mu =$ Any value

Infinite Solution: When $\lambda = 3$ and $\mu = 10$

No Solution: When $\lambda = 3$ and $\mu \neq 10$

3. Determine the value of λ for which

$x55 + 2y + 2z = 3;$

$-3y - 3z = -3;$

$-4y - 4z = 2;$

$y + z = \lambda^2$ has solution.

Ans: $\begin{bmatrix} 1 & 2 & 2 & : & 3 \\ 0 & -3 & -3 & : & -3 \\ 0 & -4 & -4 & : & 2 \\ 0 & 1 & 1 & : & \lambda^2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 2 & : & 3 \\ 0 & -3 & -3 & : & -3 \\ 0 & 0 & 0 & : & 18 \\ 0 & 0 & 0 & : & 3\lambda^2 - 3 \end{bmatrix}$

$R_4 \rightarrow 3R_4 + R_2$

$R_3 = 3R_3 - 4R_2;$

The system is inconsistent So there is no value for λ.

4. Is the given system of linear equations

$$20\,x + y - 2\,z = 17;$$

$$2\,x - 3\,y + 20\,z = 25.$$

$$3x + 20\,y - z = -18$$

are in diagonal dominant form? (Yes/NO)..

Ans: No. $\boldsymbol{|a_{22}| \ngeq |a_{21}| + |a_{23}|}$

5. Determine the value of λ for which

$x + 2y + 2\,z = 3;$

$-3y - 3\,z = -3;\ y + z = \lambda^2;$

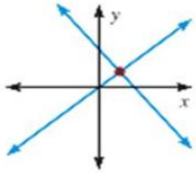
has solution.

Ans:
$$\left[\begin{array}{ccc:c} 1 & 2 & 2 & 3 \\ 0 & -3 & -3 & -3 \\ 0 & 1 & 1 & \lambda^2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc:c} 1 & 2 & 2 & 3 \\ 0 & -3 & -3 & -3 \\ 0 & 0 & 0 & 3\lambda^2 - 3 \end{array}\right]$$

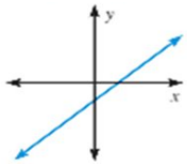
$$R_3 \to 3R_3 + R_2$$

$$3\lambda^2 - 3 = 0;\ \ \lambda = 1, -1$$

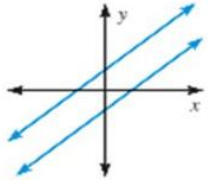
6. Specify the nature of solutions in each Graph



Graph (a)



Graph (b)



Graph (c)

a): Unique Solution **b) Infinite solution** **c) No solution.**

7. Is the statement 'A homogeneous system of equations always has a solution' true or false? Justify your answer.

Ans: Yes. Since the system is homogeneous, the trivial solution $X = 0$ (where all variables are zero) always exists.

8. Determine whether the following statement is true or false: A system of four equations with five unknown variables has a unique solution.

Ans: No

QUIZ

1. If the rank of the coefficient matrix = rank of the augmented matrix = number of unknowns, then the system of equations has:

a) No solution

b) Unique solution

c) Infinitely many solutions

d) Cannot be determined

Ans : b

2. If the rank of the coefficient matrix = rank of the augmented matrix < number of unknowns,

then the system is:

a) Consistent with infinitely many solutions

b) Inconsistent

c) Consistent with unique solution

d) Dependent and inconsistent

Ans: a

3. **Consider the system of equations:**

$$x + y + z = 6, 2x + 2y + 2z = 12, 3x + 3y + 3z = 18$$

The system has:

a) No solution

b) A unique solution

c) Infinitely many solutions

Ans : c

d) Exactly 2 solutions

4. **If the rank of a 3×3 coefficient matrix is 2 and the rank of the augmented matrix is 3, then the system of equations is:**

a) Consistent with unique solution

b) Consistent with infinitely many solutions

c) Inconsistent

Ans :c

d) Homogeneous

5. For a homogeneous system $AX = 0$, the system always has:

a)No solution

b)Only unique non-trivial solution

c) At least the trivial solution

d) Infinitely many solutions

Ans: c

6. For a system of 3 linear equations in 3 unknowns, which of the following is not possible?

a) Unique solution

b) Infinitely many solutions

c) No solution

d) Exactly 2 solutions

Ans: d

GAUSS ELIMINATION METHOD

The Gauss Elimination Method is a fundamental technique in linear algebra used to solve systems of linear equations.

Procedure:

- Write the augmented matrix of the given system of equations
- Reduce the augmented matrix to Echelon form
- Write the linear equations (starting from last) associated with the echelon form of matrix

WORKED PROBLEMS

- 1. Solve the following system by Gauss elimination method $x + y - z = 0, \quad 2x - 3y + z = -1, \quad x + y + 3z = 12, \quad y + z = 5$

Solution: $x = 1, y = 2, z = 3.$

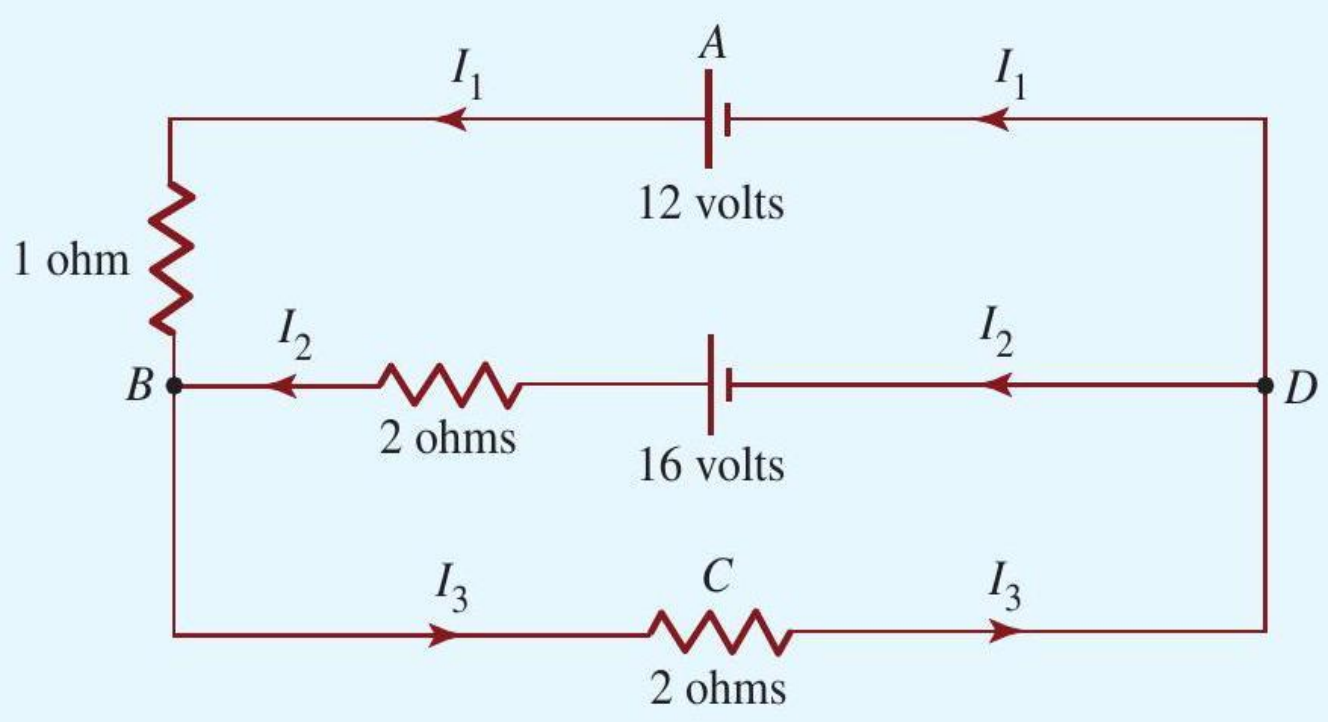
- 2. Solve the following system by Gauss elimination method

$$2x_1 - x_2 + 2x_3 = 1$$
$$-3x_1 + 4x_2 - 5x_3 = 0$$
$$x_1 + 3x_2 - 6x_3 = 0.$$

Solution:. x $= \frac{3}{7}, y = \frac{23}{21}, z = \frac{13}{21}.$

WORKED PROBLEMS

1. Determine the currents in the various branches of the electrical networks. The units of current are amps and the units of resistance are ohms.



WORKED PROBLEMS

Step 1: Construct the system of linear equations for the given circuit

Along the junction point B, apply KCL

$I_1 + I_2 = I_3 \ldots\ldots\ldots (a)$

Similarly, along the junction point D

$I_3 = I_1 + I_2 \ldots\ldots\ldots (b)$

Solving equations (a) and (b) reduces to a single linear equations given by,

$I_1 + I_2 - I_3 = 0 \ldots\ldots (1)$

Along the closed loop ABCDA apply KVL we get,

$I_1 + 2I_3 = 12 \ldots\ldots\ldots (2)$

Similarly, along the closed loop BCDB,

$2I_2 + 2I_3 = 16 \Rightarrow I_2 + I_3 = 8 \ldots\ldots (3)$

WORKED PROBLEMS

Now the system of linear equations associated with the given circuit is given by,

$$I_1 + I_2 - I_3 = 0$$
$$I_1 + 2I_3 = 12$$
$$I_2 + I_3 = 8$$

The augmented matrix for the given system of linear equations is given by,

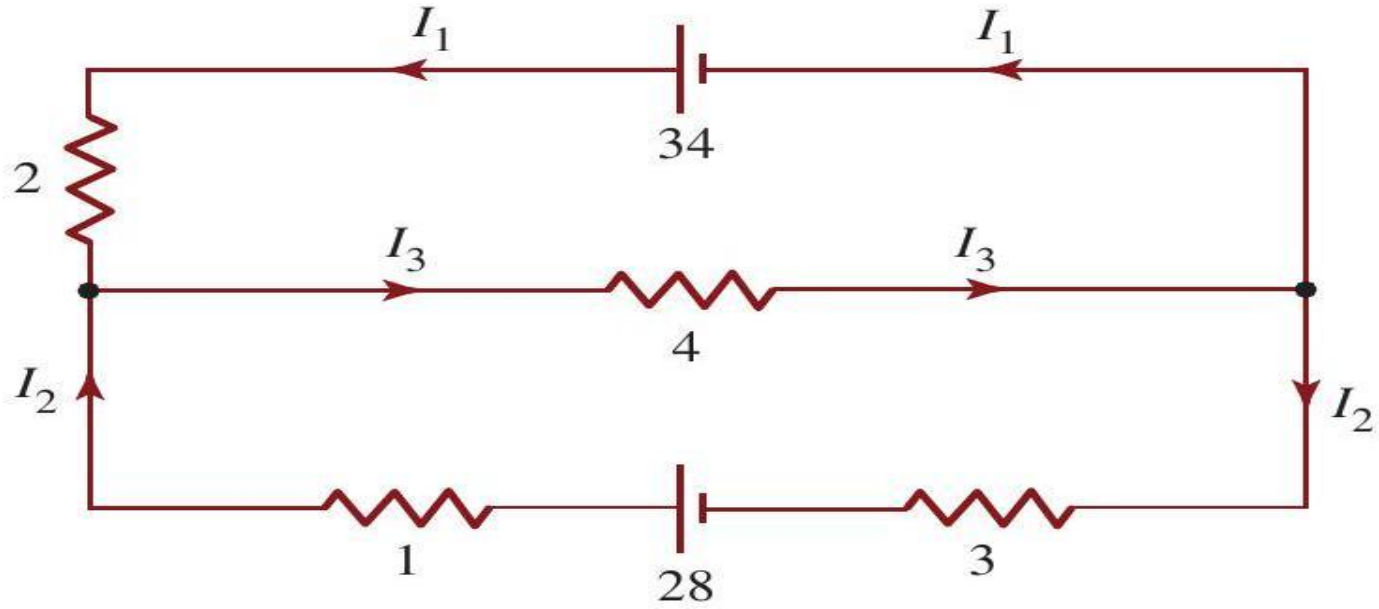
$$[A:B] = \left[\begin{array}{cccc} 1 & 1 & -1 & 0 \\ 1 & 0 & 2 & 12 \\ 0 & 1 & 1 & 8 \end{array}\right]$$

Next fallow the procedure of Gauss elimination method to find the currents I_1, I_2 and I_3.

Solution: $I_1 = 2A, I_2 = 3A$ and $I_3 = 5A$

WORKED PROBLEMS

2. Determine the currents in the various branches of the electrical networks, given by $I_1 + I_2 - I_3 = 0$, $2I_1 + 4I_3 = 34$, $I_2 + 4I_3 = 28$. Assume the current are measured in amperes (A) and resistances are measured in ohms(Ω).



Solution: $I_1 = 4.14A, I_2 = 2.29A,\ I_3 = 6.43A$

WORKED PROBLEMS

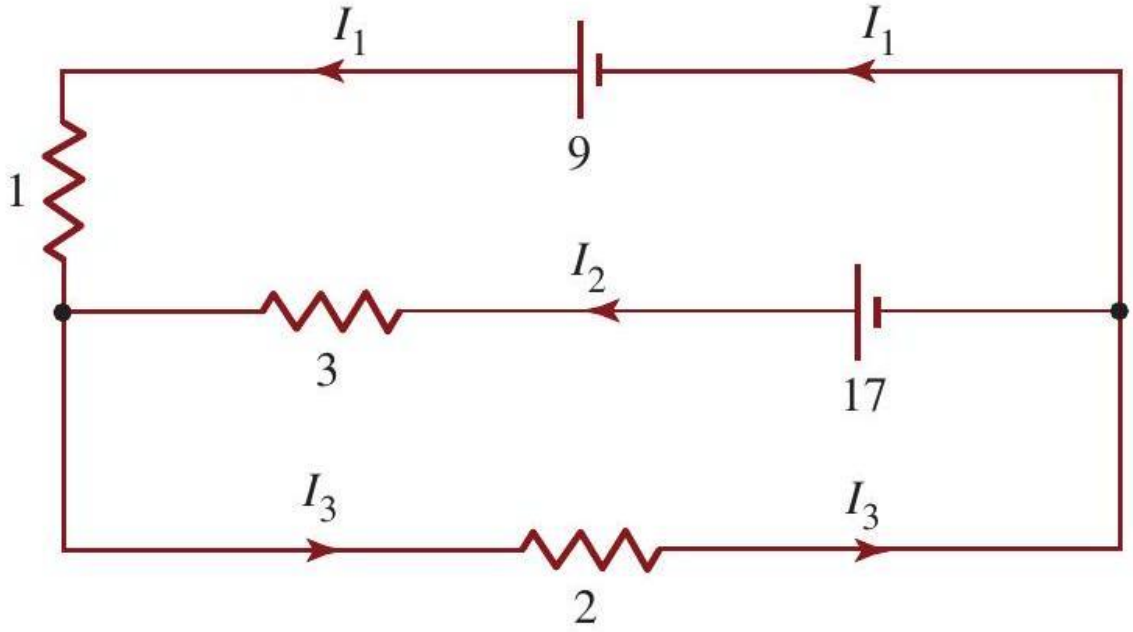
3. Alex made investments in three areas. The investment pattern and total annual interest earned are represented by the following equation:
$x + y + z = 12000, z = y + 4000$, $0.03x + 0.04y + 0.07z = 670$. Determine the amount invested in each area x , y *and* z.
Solution: $x = 2000, y = 3000, z = 7000$

4. Raj made investments across two avenues. The investment pattern and total annual return are represented by the following equations $x + y = 18000$, $0.20x + 0.06y = 2160$. Determine the amounts invested in each avenue x *and* y.
Solution: $x = 7714.29, y = 10285.71$

Practice Problems

Solve the following system of equations by Gauss Elimination method:

1. $x + 2y + z = 3,\ 2x + 3y + 2z = 5,\ 3x - 5y + 5z = 2$
2. $x + y + z = 9,\ x - 2y + 3z = 8,\ 2x + y - z = 3.$
3. $2x - 3y + 4z = 7,\ 5x - 2y + 2z = 7,\ 6x - 3y + 10z = 23$
4. Determine the currents through the various branches of the electrical network.



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Practice Problems

5. The following equations represents the investment pattern
$x + y + z = 5000, z = x + 3000$ *and*
$0.02x + 0.05y + 0.09z = 890$ Estimate the amount x , y, and z.

QUESTIONS

1. A city supplies water from **three reservoirs (R1, R2, R3)** to three areas.

- Area A gets 3 units from R1, 2 from R2, and 1 from R3, total demand = 25 units.
- Area B gets 1 unit from R1, 4 from R2, and 2 from R3, total demand = 40 units.
- Area C gets 2 unit from R1, 1 from R2, and 3 from R3, total demand = 35 units.

How much water should be supplied from each reservoir?

Ans: ·Reservoir R1 supplies ≈ 2 units

· Reservoir R2 supplies ≈ 5.2 units

· Reservoir R3 supplies ≈ 8.6 units

2. A nutritionist is preparing a **diet plan** with foods x, y, and z.

- x provides 2 units protein, 1-unit carbs, 1 unit fat.
- y provides 1 unit protein, 3 carbs, 1 fat.
- z provides 1 protein, 1 carb, 2 fats.
- The daily requirement is: 20 protein, 30 carbs, 26 fats.

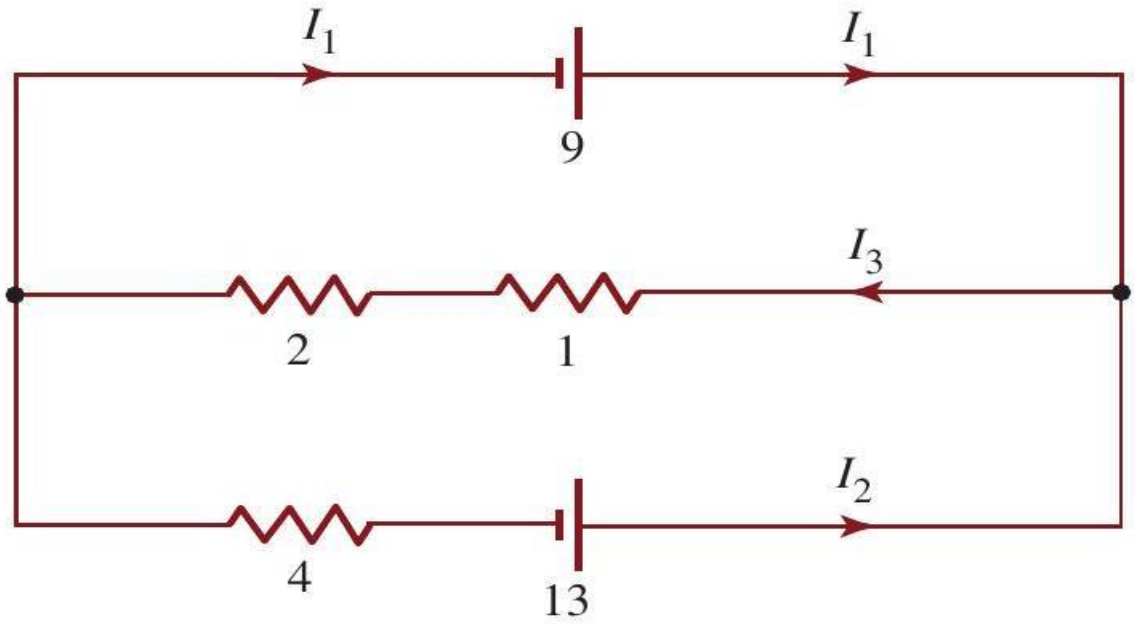
Find how many servings of each food are required by using gauss elimination method.

Ans: $\boldsymbol{x = \frac{18}{7} \approx 2.5714}$ **,** $\boldsymbol{y = \frac{44}{7} \approx 6.2857}$**,** $\boldsymbol{z = \frac{60}{7} \approx 8.5714.}$

Homework Problems

Solve the following system of equations by Gauss Elimination method:

1. $2x + y + 4z = 12,\ 4x + 11y - z = 33,\ 8x - 3y + 2z = 20.$
2. $4x + y + z = 4,\ x + 4y - 2z = 4,\ 3x + 2y - 4z = 6.$
3. $3x - y + 2z = 12,\ x + 2y + 3z = 11,\ 2x - 2y - z = 2$
4. $2x - y + 3z = 1,\ -3x + 4y - 5z = 0,\ x + 3y + 6z = 0.$
5. Determine the currents through the various branches of the electrical network.



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DIAGONALLY DOMINANT FORM

Linear system of simultaneous equations of **n** equations in **n** unknowns can be expressed as

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n = b_1$,---------(1)

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n = b_2$,---------(2)

$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots\ldots + a_{nn}x_n = b_n$--------(3)

is said to be in diagonally dominant form, if in equation (1), $|a_{11}|$ is greater the sum of the absolute values of the remaining coefficients; in (2), $|a_{22}|$ is greater than the sum of the absolute of the remaining coefficients and so on.

GAUSS-SEIDEL METHOD

The Gauss-Seidel method is an iterative method that can be used to solve a system of '*n*' linear equations in '*n*' unknowns. A starting or an initial solution is first assumed, which is then improved through successive iteration. A convergence to the actual solution is ensured if the given system of equations is arranged in the diagonally dominant form.

WORKED PROBLEMS

1. Solve the following system of equations using Gauss-Seidel method.

$$6x + 15y + 2z = 72,$$
$$x + y + 54z = 110,$$
$$27x + 6y - z = 85.$$

Solution: $[x, y, z] = [2.4255, 3.5730, 1.9259]$.

The augmented matrix for the given system of linear equation is given by,

$$\begin{bmatrix} 6 & 15 & 2 & : & 72 \\ 1 & 1 & 54 & : & 110 \\ 27 & 6 & -1 & : & 85 \end{bmatrix}$$

Check for the diagonally dominance of each row

WORKED PROBLEMS

Row 1: diagonal $|\ 6\ |$. Sum of other absolute values = $|\ 15\ | +|\ 2\ | = 17. \rightarrow 6 < 17 \rightarrow$ **not diagonally dominant**.

Row 2: diagonal $|\ 1\ |$. Sum others = $|\ 1\ | +|\ 54\ | = 55. \rightarrow 1 < 55 \rightarrow$ **not diagonally dominant**.

Row 3: diagonal $|\ -1\ | = 1$. Sum others = $|\ 27\ | +|\ 6\ | = 33. \rightarrow 1 < 33 \rightarrow$ **not diagonally dominant**.

Reorder to obtain strict diagonal dominance

We can reorder the equations to put the row with the largest diagonal elements on top. Using the order: row3, row1, row2

$$\begin{aligned} 27x + 6y - z &= 85, \\ 6x + 15y + 2z &= 72, \\ x + y + 54z &= 110. \end{aligned}$$

WORKED PROBLEMS

Gauss–Seidel iteration formulas

Solve each equation for its diagonal variable:

$$x = \frac{85 - 6y + z}{27},$$
$$y = \frac{72 - 6x - 2z}{15},$$
$$z = \frac{110 - x - y}{54}.$$

Iterations (initial guess $(x^{(0)}, y^{(0)}, z^{(0)}) = (0, 0, 0)$)

1^{st} iteration

$x(1) = 3.14814, y(1) = 3.54074, z(1) = 1.91316$

2^{nd} iteration

$x(2) = 2.43217, y(2) = 3.57204,$ z(2)=1.92584

3^{rd}iteration,

$x(3) = 2.42568,$ y(3)=3.57294, z(3)=1.92595

WORKED PROBLEMS

1. Solve the following system of equations using Gauss-Seidel method.

$$6x + 15y + 2z = 72,$$
$$x + y + 54z = 110,$$
$$27x + 6y - z = 85.$$

Solution: $[x, y, z] = [2.4255, 3.5730, 1.9259]$.

The augmented matrix for the given system of linear equation is given by,

$$\left[\begin{array}{ccc:c} 6 & 15 & 2 & 72 \\ 1 & 1 & 54 & 110 \\ 27 & 6 & -1 & 85 \end{array}\right]$$

Check for the diagonally dominance of each row

Determine whether the given system is in diagonal dominant form, if not, rearrange it accordingly.

i. $20x + y - 2z = 17, 2x - 3y + 20z = 25, 3x + 20y - z = -18$

ii. $2x + 5y + z = 12, 4x + y + z = 10, x + 2y + 6z = 15$

Practice Problems

Solve the following system of equations by Gauss Seidel Method performing 3 iterations:

1. $20x + y - 2z = 17,$
   $3x + 20y - z = -18,$
   $2x - 3y + 20z = 25.$

Solution: $[x, y, z] = [1, -1, 1]$.

2. $3x + 8y + 29z = 71,$
   $83x + 11y - 4z = 95;$
   $7x + 52y + 13z = 104.$

Solution: $[x, y, z] = [1.0579, 1.3672, 1.9617]$.

QUESTIONS

1. Solve the system of equation by Gauss seidel method performing 3 iteration

- $4x - y + z = 7$
- $x + 5y - 2z = -8$
- $2x - y + 6z = 6$

Ans: After iteration 3:

$$(x, y, z) \approx (1.22595486, -1.73121875, 0.30281192)$$

2. In an electrical network, the currents I_1, I_2, I_3 flow through three branches. Applying Kirchhoff's laws, the following equations are obtained:

- $10I_1 + 2I_2 + I_3 = 60$
- $I_1 + 8I_2 + 2I_3 = 80$
- $2I_1 + I_2 + 9I_3 = 100$

Use the Gauss–Seidel method (with initial guess $I_1 = I_2 = I_3 = 0$) and perform 3 iterations to compute the approximate currents in each branch.

Ans: After 3 iterations (approx):

$I_1 \approx 3.563299$, $I_2 \approx 7.164397$, $I_3 \approx 9.523223$

QUIZ

1. The Gauss Elimination method reduces a matrix to:

a) Diagonal form

b) Triangular (Echelon) form

c) Symmetric form

d) Identity matrix

Ans: b

2. The Gauss-Seidel method is:

a) A direct method

b) An iterative method

c) A graphical method

d) A substitution method

Ans: b

3. The Gauss-Seidel iterative method is guaranteed to converge if the coefficient matrix of the system of equations is:

a) Symmetric

b) Orthogonal

c) Diagonally dominant

d) Singular

Ans: c

4. The Gauss–Seidel method is used to solve:

a) Differential equations

b) Systems of nonlinear equations

c) Systems of linear equations

d) Determinants of matrices

Ans: c

GAUSS-SEIDEL METHOD

Home Work Problems

Solve the following system of equations by Gauss Seidel Method performing 3 iterations:

1. $20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25.$

2. $3x + 8y + 29z = 71,\ 83x + 11y - 4z = 95;\ 7x + 52y + 13z = 104.$

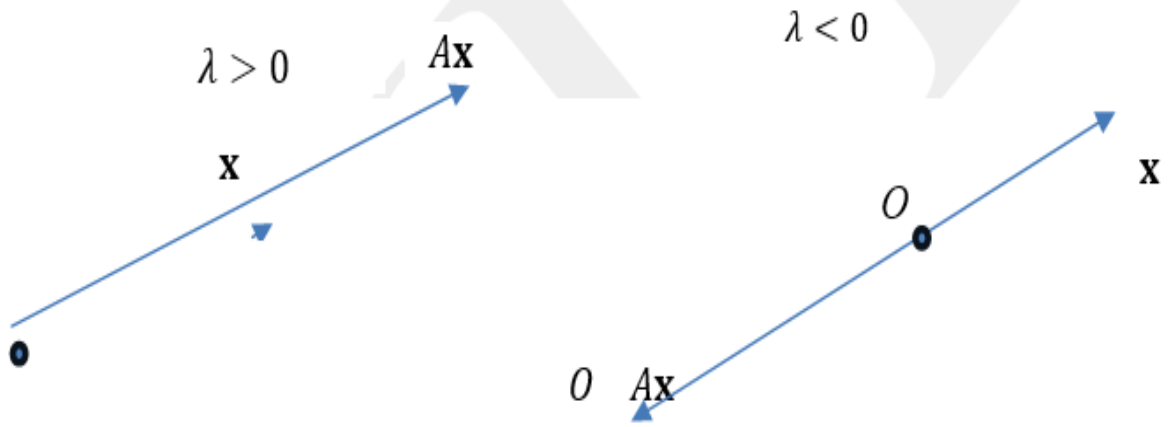
3. $10x + 2y + z = 9,\ x + 10y - z = -22,\ -2x + 3y + 10z = 22.$

4. $5x - y = 9,\ x - 5y + z = -4,\ y - 5z = 6$ taking $\left(\frac{9}{5}, \frac{4}{5}, \frac{6}{5}\right)$ as first approximation.

EIGEN VALUES AND EIGEN VECTORS

Let A be an $n \times n$ matrix. A scalar λ is called an **eigenvalue of *A*** if there exist a nonzero vector x in$\mathbb{R}^n$ such that $Ax = \lambda x$. The vector x is called **eigenvector** corresponding to λ.

Let us look at the geometrical significance of an eigenvector that corresponds to a nonzero eigenvalue. The vector Ax is in the same or opposite direction as x, depending on the sign of λ. An eigenvector of A is thus a vector whose direction is unchanged or reversed when multiplied by A.



x is an eigenvector of A. A**x** is in the same or opposite direction as **x.**

Computation of Eigenvalues and Eigenvectors

Let A be an $n \times n$ matrix with eigenvalue λ and corresponding eigenvector $\mathbf{x}$. Thus $Ax = \lambda x$.This equation may be rewritten,

$$Ax - \lambda x = 0$$

$$(Ax - \lambda I_n)x = 0$$

This matrix equation represents a system of homogeneous linear equations having matrix of coefficients $(A - \lambda I_n)x = 0$ is a solution to this system. However, eigenvectors have been defined to be nonzero vectors. Further, nonzero solutions to this system of equations can only exist if the matrix of coefficients is singular, $| A - \lambda I_n | = 0$. Hence, solving the equation $| A - \lambda I_n | = 0$ for λ leads to all the eigenvalues of A.

On expanding the determinant $| A - \lambda I_n |$ we get a polynomial of degree n in λ. This polynomial is called the **characteristic polynomial** of A. The equation $| A - \lambda I_n |$ is called the **characteristic equation** of A. This characteristic equation will have n roots, some possibly repeated,some possibly complex numbers.

Thus an $n \times n$ matrix will have n eigenvalues, some of which may be repeated, and some of which may be complex numbers. The eigenvalues are then substituted back into the equation $(Ax - \lambda I_n)x = 0$ to find the corresponding eigenvectors.

Computation of Eigenvalues and Eigenvectors

Example 1: Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$.

Solution: Let us first derive the characteristic polynomial of A. We get,

$$A - \lambda I_2 = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 - \lambda & -6 \\ 3 & 5 - \lambda \end{bmatrix}$$

$$|A - \lambda I_2| = (-4 - \lambda)(5 - \lambda) + 18 = \lambda^2 - \lambda - 2$$

$$\text{i.e.,} \quad \lambda^2 - \lambda - 2 = 0$$

$$(\lambda - 2)(\lambda + 1) = 0$$

$$\lambda = 2 \text{ or } -1.$$

The eigenvalues of A are 2 or -1.

The corresponding eigenvectors are found by using these values of in the equation $(A - \lambda I_2)x = 0$. There are many eigenvectors corresponding to each eigenvalue.

Computation of Eigenvalues and Eigenvectors

When $\lambda = 2$, we solve the equation $(A - 2I_2)\,x = 0$ for x . The matrix $(A - 2I_2)$ is obtained by subtracting 2 from the diagonal elements of A. We get,

$$\begin{bmatrix} -6 & -6 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

This leads to the system of equation, $-6x_1 - 6x_2 = 0$

$$3x_1 + 3x_2 = 0$$

giving $x_1 = -x_2$. The solutions to this system of equations are $x_1 = -r$, $x_2 = r$,where r is a scalar. Thus, the eigenvectors of A corresponding to $\lambda = 2$ are nonzero vectors of the form

$$r \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 0$$

Computation of Eigenvalues and Eigenvectors

When $\lambda = -1$, we solve the equation $(A + 1I_2)x = 0$ for x . The matrix $(A + 1I_2)$ is obtained by subtracting 1 from the diagonal elements of A. We get,

$$\begin{bmatrix} -3 & -6 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

This leads to the system of equation, $-3x_1 - 6x_2 = 0$

$$3x_1 + 3x_2 = 0$$

giving $x_1 = -2x_2$ .The solutions to this system of equations are $x_1 = -2s$, $x_2 = s$ where s is a scalar. Thus, the eigenvectors of A corresponding to $\lambda = -1$ are nonzero vectors of the form,

$$s\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 0$$

Key Ideas

- For a square matrix A, a nonzero vector $v$is an **eigenvector** if

$$Av = \lambda v$$

- where $\lambda$is the **eigenvalue**.
- Eigenvalues are found from the **characteristic equation**:

Computation of Eigenvalues and Eigenvectors

Example 2: Find the eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Solution:

We first obtain the matrix $[A - \lambda I_3]$.

$$A - \lambda I_3 = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

The characteristic equation is $|A - \lambda I_3| = 0$.

Using row and column operations to simplify determinant, we obtain

$$|A - \lambda I_3| = \begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 3-\lambda\,[(2-\lambda)(-5-\lambda)+6]$$

OR $|A - \lambda I_3| = -\lambda^3 + 13\lambda - 12 = 0$

Solving the equation, $-\lambda^3 + 13\lambda - 12 = 0$ gives the eigen values as

$\lambda = 1, 3, -4$.

1. Define an eigenvalue and eigenvector of a matrix
2. Find eigenvalue of the following matrices

(i) $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Practice Problems

Find Eigen value and corresponding Eigen vector of the folllowing matrices :

$A = \begin{bmatrix} 2 & -3 \\ 2 & -5 \end{bmatrix}$; $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$; $A = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$; $A = \begin{bmatrix} 2 & 0 \\ 5 & 3 \end{bmatrix}$

Homework Problems

Determine the eigenvalues and eigen vectors of the matrices

a) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ *b)* $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ *c)* $\begin{bmatrix} 5 & 6 \\ -2 & -2 \end{bmatrix}$ *d)* $\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$ *e)* $\begin{bmatrix} 5 & 2 \\ -8 & -3 \end{bmatrix}$

Reinforcement (Review) Questions:

Q1. If v is an eigenvector of $A$with eigenvalue λ, then

a) $Av = v$ b) $Av = \lambda v$ c) $Av = 0$ d) $Av = A$

Answer: b) $Av = \lambda v$

Q2. The eigenvalues of the identity matrix $I_n$are:

a) All zeros b) All ones c) All negative d) None

Answer: b) All ones

Q3. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$,then its eigenvalues are:

a) 2 and 3 b) 0 and 1 c) 5 and -1 d) None

Answer: a) 2 and 3

Q4. The sum of eigenvalues of a square matrix equals:

a) The product of diagonal entries b) The rank of the matrix

c) The trace of the matrix d) Always 0

Answer: c) The trace of the matrix

Q5. The product of eigenvalues of a matrix equals:

a) The determinant of the matrix b) The sum of the diagonal entries

c) The rank d) Zero always

Answer: a) The determinant of the matrix

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Reinforcement (Review) Questions:

1.A nonzero vector $v$satisfying $Av = \lambda v$is called an _____.
Answer: Eigenvector

2.The scalar λin $Av = \lambda v$is called an _____.
Answer: Eigenvalue

3.The equation is called the _____ **equation**.
Answer: Characteristic equation

4.The sum of the eigenvalues of a matrix equals the _____ of the matrix.
Answer: Trace

5.The product of the eigenvalues of a matrix equals the _____ of the matrix.
Answer: Determinant

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