26BSCC102

MANGALORE INSTITUTE OF TECHNOLOGY & ENGINEERING

(A Unit of Rajalaxmi Education Trust®, Mangalore)
Autonomous Institute affiliated to VTU, Belagavi, Approved by AICTE, New Delhi
Accredited by NAAC with A+ Grade & ISO 9001:2015 Certified Institution

Model Question Paper
First Semester B.E Degree Examination

Linear Algebra & Differential Equations

Time: 3 Hours **Max. Marks: 100**

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M: Marks, L: RBT (Revised Bloom's Taxonomy) level, C: Course outcomes.

		Module -1	M	L	C
Q1	a.	i) Define Row Echelon form of a matrix. ii) Determine whether the given augmented matrix is consistent: $\begin{bmatrix} 2 & 4 & 6: & 6 \\ 0 & 0 & 6: & 4 \\ 0 & 4 & 5: & 8 \end{bmatrix}$	3 3	L1 L2	CO1 CO1
	b.	Solve the following system of equations by Gauss-Elimination method: $2x_1 - x_2 + 2x_3 = 1, \quad -3x_1 + 4x_2 - 5x_3 = 0, \quad x_1 + 3x_2 - 6x_3 = 0$	7	L2	CO2
	c.	Determine the currents through the various branches of the electrical network by Gauss-Seidel method, correct to three decimal places, given the system of equations is: $2I_1 + I_2 + 9I_3 = 100, 10I_1 + 2I_2 + I_3 = 60, I_1 + 8I_2 + 2I_3 = 80$(Perform 3 iterations).	7	L3	CO2
		OR			
Q2	a.	i) Define Rank of a matrix. Find the rank of the matrix $A = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ii) Determine whether the given system of equations are diagonally dominant, if not, rearrange it accordingly: $2x - 3y + 20z = 25, 20x + y - 2z = 17, 3x + 20y - z = -18$	3 3	L1 L2	CO1 CO1
	b.	Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$.	7	L2	CO2
	c.	The following equations represent Alex's investment pattern in Mutual funds (x), ffixed deposits (y) and corporate bonds(z): $x + y + z = 12000, \ z = y + 4000, \ 0.03x + 0.04y + 0.07z = 670$. Determine the amounts invested in x, y *and* z by Gauss Elimination method.	7	L3	CO2
		Module- 2			
Q3	a.	i) Define linearly independent and linearly dependent vectors. ii) Express $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ *and* $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$	2 4	L1 L2	CO1 CO1
	b.	Consider the vectors: $u = [1 \ 2 \ 4]^T, v = [2 \ -3 \ 5]^T, w = [4 \ 2 \ -3]^T$ in $\mathbb{R}^3$. i) Find: $\langle u, v \rangle$, $\langle v, w \rangle$, $\langle u, u \rangle$, $\|v\|$ *and* $\|w\|$ ii) Determine whether vector w is orthogonal to u and v.	7	L2	CO2
	c.	A dataset containing customer reviews of a product consists of 40 positive reviews, 35 negative reviews, and 15 neutral reviews. The corresponding sentiment labels are assigned weights of $(0.5, 0.5, 0)$. Assume that the reviews are classified as POSITIVE	7	L3	CO2

		(1) and NEGATIVE (0), where positive reviews are assigned a value of 1 and negative reviews are assigned a value of 0. i) Name the mathematical concept involved in calculating the weighted sum. ii) Evaluate the weighted sum using the given data and classify the overall sentiment as POSITIVE or NEGATIVE.			
		OR			
Q4	a.	i) Define the Vector Space**.** ii) Check whether the set $M = \left\{\begin{bmatrix} a & b \\ c & 1 \end{bmatrix} \mid a, b, c \in \mathbb{R}\right\}$ form a vector space over $\mathbb{R}$.	2 4	L1 L2	CO1 CO1
	b.	Examine the vectors $[1\ 1\ 2\ 4]^T, [2\ -1\ -5\ 2]^T, [1\ -1\ -4\ 0]^T$ and $[2\ 1\ 1\ 6]^T$ for linearly dependence in $\mathbb{R}^4$ and extract a linearly independent subset.	7	L2	CO2
	c.	The current direction of a drone is given by the vector $u = [5\ 3\ 3]^T$. The drone needs to adjust its movement to be as close as possible to a specific direction represented by the vector $v = [2\ 2\ 0]^T$. i) Find the projection of u onto v, to determine the closest alignment of the drone's direction with the target direction ii) Obtained projection matrix P and Show that the obtained projection matrix P is symmetric and idempotent.	7	L3	CO2
		Module – 3			
Q5	a.	i) Define Kernel and Range of a linear transformation $T: U \to V$. Write their mathematical representations. ii) Determine the transformation matrix for the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$, defined by $T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x + y \\ 3y - z \end{bmatrix}$ w.r.t. to the standard basis.	3 3	L1 L2	CO1 CO1
	b.	Determine whether the following transformations are linear: i) $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x + y, x, 3x - y)$ ii) $T: \mathbb{R}^1 \to \mathbb{R}^3$ defined by $T(x) = (x, x^2, x^3)$	7	L2	CO2
	c.	Let the vectors $u_1 = [1\ 2\ 0]^T$, $u_2 = [1\ 3\ 2]^T$, $u_3 = [0\ 1\ 3]^T$ form a basis S of $\mathbb{R}^3$. i) Find the change of basis matrix P from the standard basis E of $\mathbb{R}^3$ to the basis S. ii) Find the change of basis matrix Q from the basis S to the standard basis E.	7	L3	CO2
		OR			
Q6	a.	i) State the Rank–Nullity Theorem for a linear transformation. Explain the terms Rank and Nullity appearing in the theorem. ii) Represent the transformation $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$ geometrically and specify its nature.	3 3	L1 L2	CO1 CO1
	b.	Determine the kernel and range of the transformation defined by the following matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 4 \end{bmatrix}$.	7	L2	CO2
	c.	Apply the transformation matrix $A = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$ to each of the coordinates $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and find its transformation. Represent it graphically and specify its nature.	7	L3	CO2
		Module – 4			
	a.	i) Identify the order and degree of the given Differential Equation:	2	L1	CO1

Q7		$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} \cdot \frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^3 = 0$ ii) Solve: $\frac{dy}{dx} + 2y = e^{-2x}$.	4	L2	CO1
	b.	Use the Runge-Kutta method of fourth order with $h = 0.2$ to obtain an approximate solution of the initial value problem $(y^2 - x^2)\,dx = (y^2 + x^2)dy$, $y\,(0) = 1$ at $y\,(1.2)$, correct to 4 decimal places.	7	L2	CO3
	c.	A certain population of Lactobacillus bacteria increases continuously at a rate that is proportional to its present number. The initial population of the bacterial culture is 140, it jumped to 720 bacteria in 4 hours. How many bacteria will be present in 10 hours? How long will it take the population to double?	7	L3	CO4
		OR			
Q8	a.	i) Define a Linear Differential Equation. ii) Solve the initial value problem $\frac{dy}{dx} = 2x$ with $y(0) = 3$.	2 4	L1 L2	CO1 CO1
	b.	Apply Modified Euler's method for the initial value problem: $\frac{dy}{dx} = x^2 + y$, $y\,(0) = 1$, find $y\,(0.1)$ by taking $h = 0.1$, correct to 4 decimal places.	7	L2	CO3
	c.	An LR circuit consists of an e.m.f. $E(t) = 24$, a resistor $8\,\Omega$, an inductor $0.5\,H$ and that the current is initially zero. What is the current at $t = 2\,min$. Find maximum current attained by the circuit.	7	L3	CO4
		Module – 5			
Q9	a.	i) Given the general solution $y = C_1 cos2x + C_2 sin2x - \frac{1}{5}sin3x + \frac{x}{4}sin2x$ • Identify the complementary function (C.F.) of the given solution. • Identify the nature of the roots and write the roots. Solve: $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$	3 3	L1 L2	CO1 CO1
	b.	Apply the Milne's Predictor-Corrector method to compute $y\,(0.8)$ given that $y'' = 1 - 2yy'$, correct to 4 decimal places.	7	L2	CO3
	c.	A $5\,kg$ object is attached to spring and will stretch the spring $0.6125\,m$ by itself. There is no damping in the system and a forcing function of the form $F(t) = 15cos(4t)$ attached to the object. The object is initially displaced $0.3\,m$ downward from its equilibrium position and given a velocity of 0.2m/sec. Find the displacement at any time t. Interpret the general solution in the absence of external force.	7	L3	CO4

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
$y' = z$	0	0.1996	0.3972	0.5689

		OR			
Q10	a.	i) Formulate Linear Differential Equation and write the type of Oscillation: Given $m = 5kg$, $k = 45N/m$, $c = 0$ and the external force is $f(t) = 20cos3t$. ii) Solve: $y'' - 4y' + y = 0$	3 3	L1 L2	CO1 CO1
	b.	Solve $(D^2 + 16)y = e^{-3x} + cos4x$.	7	L2	CO3
	c.	An electric circuit consists of an inductance of $0.1H$ a resistance of 20Ω and a condenser of $25\ \mu F$. Find the charge q at any time t, given that $q(0) = 0.05$ and $i(0) = 0$.	7	L3	CO4

*****-