

Module 3: Principles of Force System in Civil Engineering	No. of Hrs: 06
Force System: Force, Moment of force, Concept of idealization, System of forces, Principle of physical independence of forces, Principle of Superposition of forces, Principle of transmissibility of forces, Resolution of a force and Composition of forces, Analysis of Coplanar Concurrent Force System Self-study component: Explore real-life applications of the moment of force. Identify practical examples of force system.	

3.1 INTRODUCTION TO ENGINEERING MECHANICS

Mechanics is the physical science concerned with the study of response of bodies under the application of forces. Engineering mechanics is the application of mechanics to the solution of engineering problems. It is broadly classified into three types:

- Mechanics of rigid bodies**
- Mechanics of deformable bodies**
- Mechanics of fluids.**

Mechanics of Rigid Bodies

It is the branch of science which deals with the study of bodies that do not undergo any deformation under the application of forces. It can further be classified into Statics and Dynamics.

Statics

It is the branch of mechanics which deals with the study of the behaviour of bodies or particles in the state of rest.

Dynamics

It is the branch of mechanics which deals with the study of the behaviour of bodies or particles in the state of motion. Dynamics is further divided into two types:

- **Kinematics:** The forces causing the motion are not considered.
- **Kinetics:** The forces causing the motion are mainly considered.

3.2 FORCE

Force: It is the external agency which tends to change the state of a body or a particle. When a force is applied to a body which is at rest, the body may remain in the state of rest or it may move with some velocity. The SI unit of force is newton.

Basic Terms Related to Force

Mass (m): The total amount of matter present in a body is known as its mass. The unit of mass is the kilogram, abbreviated kg.

Weight: A body is attracted towards the earth due to gravitation. This causes an acceleration directed towards the centre of earth. It is called acceleration due to gravity and is denoted by g . The resulting force is equal to the weight of body.

$$\text{Weight} = \text{mass} \times \text{acceleration due to gravity}$$

$$W = mg, \text{ in newton, where } g = 9.81 \text{ m/s}^2$$

Scalar quantity: A physical quantity which has only magnitude, is called scalar quantity. For example, time, mass, density, volume, distance, and so forth.

Vector quantity: A physical quantity which has a direction in addition to magnitude, is known as vector quantity. For example, force, displacement, velocity, acceleration, and so forth.

Newton's Laws of Motion

Newton's First law: This law states that 'every body continues in its state of rest or of uniform motion along a straight line, so long as it is under the influence of a balanced force system'.

Newton's Second law: This law states that 'the rate of change momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it'.

Newton's Third law: This law states that 'action and reaction are equal in magnitude but opposite in direction'.

Elements of a Force or Characteristics of a Force

A force can be identified by its four characteristics:

- (i) **Magnitude:** The length of the vector represents the magnitude of force, as shown in Figure 3.1.

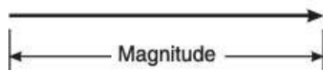


Figure 3.1: Magnitude of force vector.

- (ii) **Direction:** The direction of a force can be represented by an arrowhead.
- (iii) **Line of action:** It is the line along which the force acts.
- (iv) **Point of application:** It is the point at which the force acts.

3.3 MOMENT OF A FORCE

The turning effect produced by a force on a body is known as the moment of the force. The magnitude of the moment is given by the product of the magnitude of the force and the perpendicular distance between the line of action of the force and the point or axis of rotation. This is shown in Figure 3.2(a).

Types of moments

- (i) If the tendency of a force is to rotate the body in the clockwise direction, it is said to be a clockwise moment and is taken positive, as shown in Figure 3.2(b).
- (ii) If the tendency of a force is to rotate the body in the anticlockwise direction, it is said to be anticlockwise moment and is taken negative as shown in Figure 3.2(c).

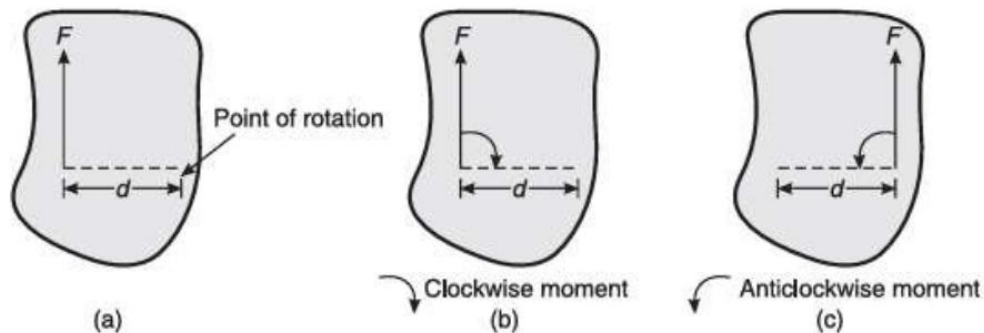


Figure 3.2: Moment of a force.

3.4 CONCEPT OF IDEALIZATION

In engineering mechanics, actual bodies and loads are represented by simplified models (idealizations) so that they can be analysed easily. The idealizations commonly used are:

Particle: A body of infinitely small volume whose mass can be neglected, is called a particle.

Body: The assemblage of a number of particles is known as a body.

Rigid body: A rigid body is one in which the positions of the constituent particles do not change under the application of external forces, such as the position of particles 1 and 2 in Figure 3.3.

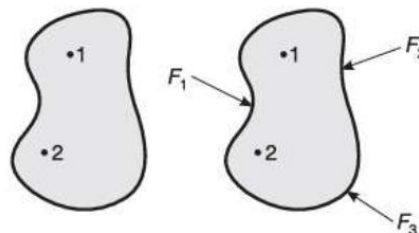


Figure 3.3: Rigid body.

Deformable body: A deformable body is one in which the positions of constituent particles change under the application of external forces, such as the positions of particles 1 and 2 in Figure 3.4.

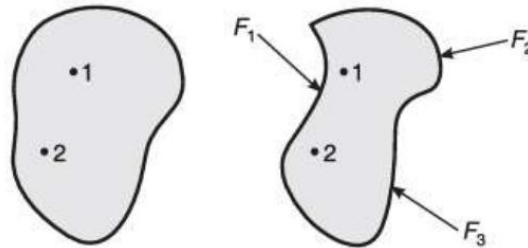


Figure 3.4: Deformable body.

Continuum: A continuous distribution of molecules in a body without intermolecular space is called the continuum.

Point force: A force which is acting at a fixed point is known as the point force. Let us consider a man climbing a ladder. The weight of the man is not actually concentrated at a fixed point but for the purpose of analysis it is assumed to be concentrated at a particular point.

3.5 SYSTEM OF FORCES

If two or more forces are acting on a body or a particle, then it is said to be a force system, such as that shown in Figure 3.5.

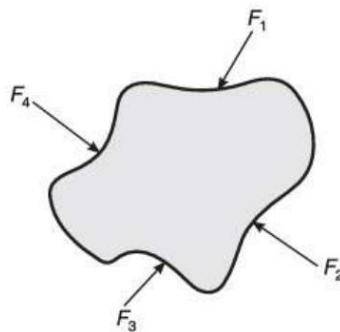


Figure 3.5: A force system.

Types of Force System

The types of force system are:

1. Coplanar force system
2. Non-coplanar force system
3. Collinear force system.

Coplanar force system

If two or more forces are acting in a single plane, then it is said to be a coplanar force system. The types of coplanar force system are:

- (i) Coplanar concurrent force system
- (ii) Coplanar non-concurrent force system
- (iii) Coplanar parallel force system.

If two or more forces are acting in a single plane and their lines of action pass through a single point, then it is said to be a **coplanar concurrent force system**. See Figure 3.6.

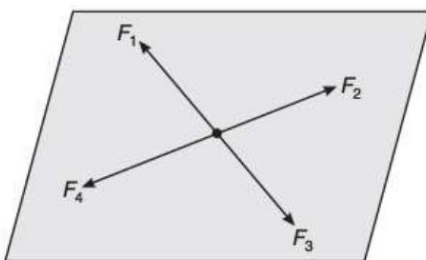


Figure 3.6: Coplanar concurrent force system.

If two or more forces are acting in a single plane and their lines of action do not meet at a common point, then the forces constitute a **coplanar non-concurrent force system**. See Figure 3.7.

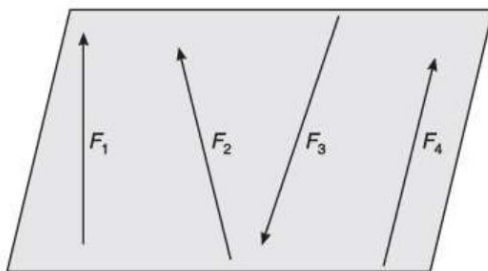


Figure 3.7: Coplanar non-concurrent force system.

If two or more forces are acting in a single plane with their lines of action parallel to one another, then it is said to be a **coplanar parallel force system**. The coplanar parallel force system is of two types:

- (i) **Like parallel force system:** All the forces act parallel to one another and are in the same direction, as shown in Figure 3.8.

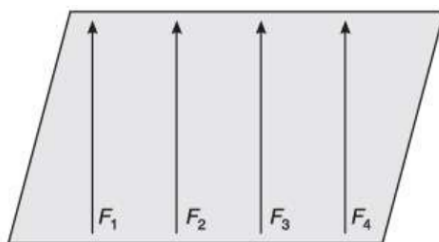


Figure 3.8: Like parallel force system.

- (ii) **Unlike parallel force system:** The forces act parallel to another, but some of the forces have their line of action in opposite directions, as shown in Figure 3.9.

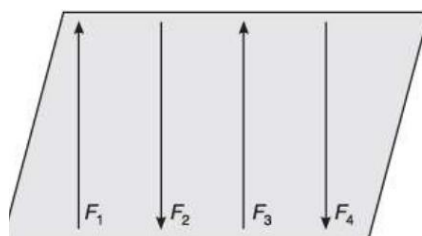


Figure 3.9: Unlike parallel force system.

Non-coplanar force system

If two or more forces are acting in different planes, the forces constitute a non-coplanar force system. Such a system of forces can be

- (i) Non-coplanar concurrent force system
- (ii) Non-coplanar non-concurrent force system
- (iii) Non-coplanar parallel force system.

If a system has two or more forces acting on different planes but pass through the same point, then it is said to be a **non-coplanar concurrent force system**. See Figure 3.10.

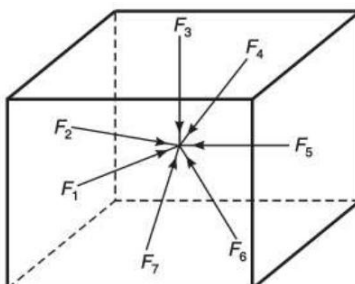


Figure 3.10: Non-coplanar concurrent force system.

If two or more forces are acting on different planes but do not pass through the same point, they constitute a **non-coplanar non-concurrent force system**. See Figure 3.11.

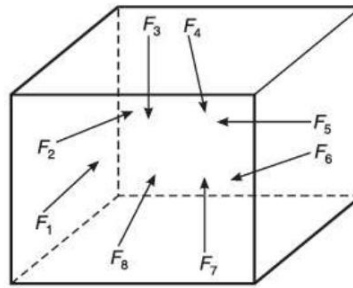


Figure 3.11: Non-coplanar non-concurrent force system.

If two or more forces are acting in different planes and are parallel to one another, the system is said to be a **non-coplanar parallel force system**. See Figure 3.12.

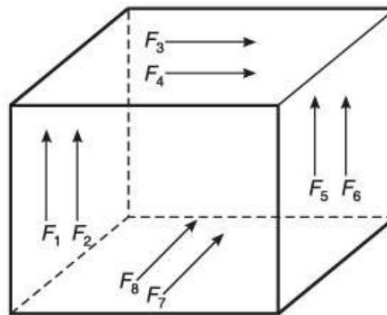


Figure 3.12: Non-coplanar parallel force system.

Collinear force system

If the lines of action of two or more forces coincide with one another, it is called a collinear force system as shown in Figure 3.13.

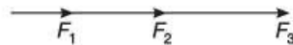


Figure 3.13: Collinear force system.

Non-collinear force system

If the lines of action of the forces do not coincide with one another, it is called a non-collinear force system as shown in Figure 3.14.

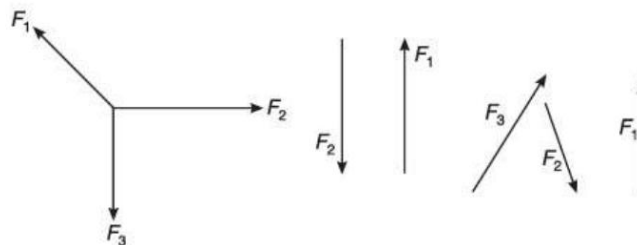


Figure 3.14: Non-collinear force system.

3.6 PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES

This principle states that the action of a force on a body is not affected by the action of any other forces acting on the body. Each force produces its own effect on the body as if the other forces were absent.

3.7 PRINCIPLE OF SUPERPOSITION OF FORCES

This principle states that the net effect of a system of forces on a body is same as that of the combined effect of individual forces on the body.

3.8 PRINCIPLE OF TRANSMISSIBILITY OF FORCES

This principle states that a force can be transmitted from one point to another point along the same line of action such that the effect produced by the force on a body remains unchanged. Let us consider a rigid body subjected to a force of F at point O as shown in Figure 3.15. According to the principle of transmissibility, the force F can be transmitted to a new point O' along the same line of action such that the net effect remains unchanged.

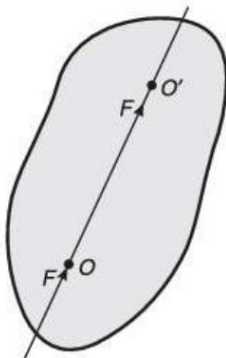


Figure 3.15: Transmissibility of force F from point O to O' .

3.9 RESOLUTION OF A FORCE AND COMPOSITION OF FORCES

Resolution of a Force

The process of splitting of a force into its two rectangular components (horizontal and vertical) is known as resolution of the force, as shown in Figure 3.16. In this figure, F is the force which makes an angle θ with the horizontal axis, and has been resolved into two components, namely F_x and F_y , along the x -axis and y -axis respectively.

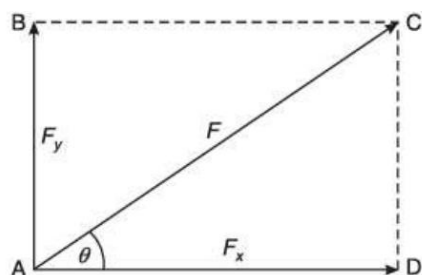


Figure 3.16: Resolution of a force.

In ΔCAD ,

$$\cos \theta = F_x / F \Rightarrow F_x = F \cos \theta$$

$$\sin \theta = F_y / F \Rightarrow F_y = F \sin \theta$$

If, on the other hand, θ is the angle made by the force F with the vertical axis, then

$$F_y = F \cos \theta ; \quad F_x = F \sin \theta$$

Note: If the force F makes an angle of θ with the horizontal, the horizontal component of the force is $F \cos \theta$.

Composition of Forces

It is the process of combining a number of forces into a single force such that the net effect produced by the single force is equal to the algebraic sum of the effects produced by the individual forces. The single force in this case is called the **resultant force** which produces the same effect on the body as that produced by the individual forces acting together. For example, in Figure 3.17,

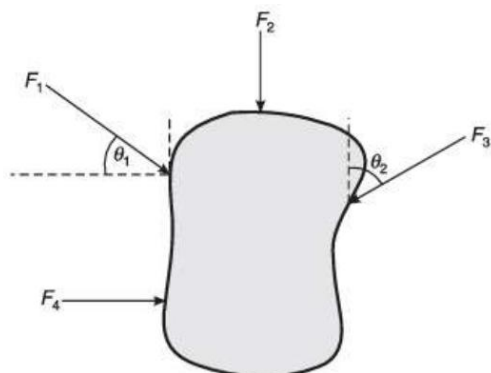


Figure 3.17: A body acted upon by a number of forces.

ΣF_x = algebraic sum of the components of the forces along the x -axis

i.e. $\Sigma F_x = F_4 + F_1 \cos \theta_1 - F_3 \sin \theta_2$

and ΣF_y = algebraic sum of the components of the forces along the y -axis

i.e. $\Sigma F_y = -F_2 - F_1 \sin \theta_1 - F_3 \cos \theta_2$

Note: The positive and negative convention of forces used in the resolution of forces in Figure 3.16 is as that shown in Figure 3.18.

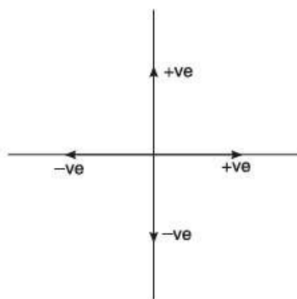


Figure 3.18: Positive and negative convention of forces.

∴ The magnitude of the resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and the direction of the resultant,

$$\theta = \tan^{-1} (\Sigma F_y / \Sigma F_x)$$

3.10 ANALYSIS OF COPLANAR CONCURRENT FORCE SYSTEM

If two or more forces are acting in a single plane and passing through a single point, such a force system is known as coplanar concurrent force system.

In a coplanar concurrent force system, we can calculate the magnitude and direction of the resultant. The position, however, cannot be determined because all forces are meeting at a common point. Thus,

The magnitude of resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

Direction of resultant,

$$\theta = \tan^{-1} (\Sigma F_y / \Sigma F_x)$$

The steps to solve problems in the coplanar concurrent force system are, therefore, as follows:

1. Calculate the algebraic sum of all the forces acting in the x -direction (i.e. ΣF_x) and also in the y -direction (i.e. ΣF_y).
2. Determine the magnitude of the resultant using the formula, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$.
3. Determine the direction of the resultant using the formula, $\theta = \tan^{-1} (\Sigma F_y / \Sigma F_x)$.

Free Body Diagram

A free body diagram is a sketch of a body, a portion of a body, or two or more bodies completely isolated or free from all other bodies, showing the forces exerted by all other bodies on the one being considered. Characteristics of free body diagram:

- It is a diagram or sketch of a body.
- The body is shown completely separated from all other bodies.
- The action on the body of each body removed in the isolating process is shown as a force or forces on the diagram.

A free-body diagram is a crucial tool in physics and engineering for analyzing forces and motion. By isolating a single object or system and showing all the external forces acting on it, you can apply Newton's Laws to solve for unknown forces or accelerations.

Example of a Free-Body Diagram

Consider a block resting on a table (Figure 3.19).

1. **Isolate the body:** The "body" is the block itself.
2. **Identify forces:**
 - **Weight (W or mg):** The force of gravity pulls the block down. This force acts on the block from the Earth.
 - **Normal Force (N):** The table pushes up on the block, preventing it from falling through. This is a contact force.
3. **Draw the diagram:** draw a simple box (representing the block) and then draw arrows originating from it to represent each force.

In this simple example, since the block is not moving vertically, the forces are balanced: the upward normal force (N) is equal in magnitude to the downward weight (W). This is an application of Newton's First Law (for an object in equilibrium) or Second Law ($\Sigma F_y = 0$).

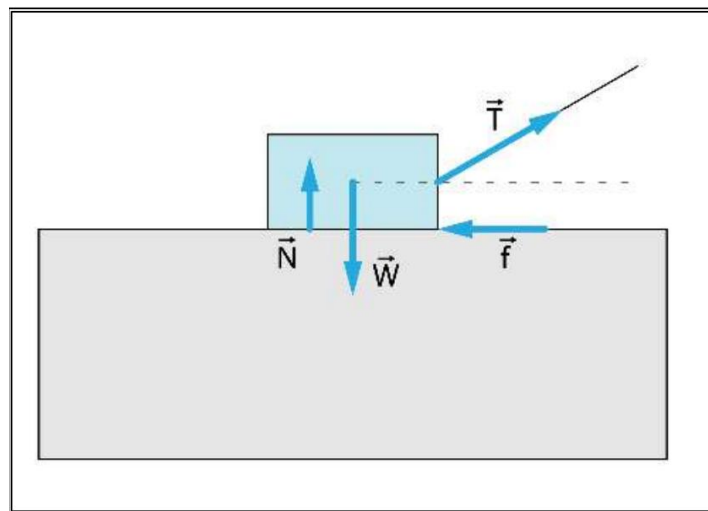


Figure 3.19: Free body diagram.

Solved Problems on Coplanar Concurrent Force System

Problem 1: A force of 200 N is acting at a point making an angle of 40° with the horizontal (Figure 3.20). Determine the components of this force along the x and y directions.

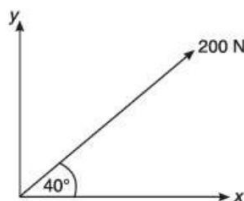


Figure 3.20: Problem 1.

Solution Component along the x -direction,

$$F_x = F \cos \theta = 200 \times \cos 40^\circ = 153.208 \text{ N}$$

Ans.

Component along the y -direction,

$$F_y = F \sin \theta = 200 \times \sin 40^\circ = 128.557 \text{ N}$$

Ans.

Problem 2: Five coplanar forces are acting at a point as shown in Figure 3.21. Determine the resultant in magnitude and direction.

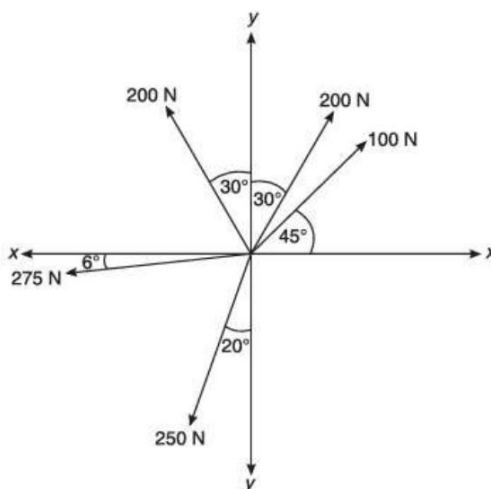


Figure 3.21: Problem 2.

Solution Here:

$$\Sigma F_x = -200 \sin 30^\circ - 275 \cos 6^\circ - 250 \sin 20^\circ + 100 \cos 45^\circ + 200 \sin 30^\circ = -288.287 \text{ N}$$

$$\Sigma F_y = 200 \cos 30^\circ - 275 \sin 6^\circ - 250 \cos 20^\circ + 100 \sin 45^\circ + 200 \cos 30^\circ = 153.452 \text{ N}$$

$$\therefore R = \sqrt{(-288.287)^2 + (153.452)^2} = 326.584 \text{ N}$$

Ans.

$$\text{Also, } \theta = \tan^{-1} (\Sigma F_y / \Sigma F_x)$$

$$\text{or } \theta = \tan^{-1} (153.452 / -288.287)$$

$$\therefore \theta = 28.02^\circ$$

Ans.

Problem 3: Find the resultant of the coplanar concurrent force system shown in Figure 3.22.

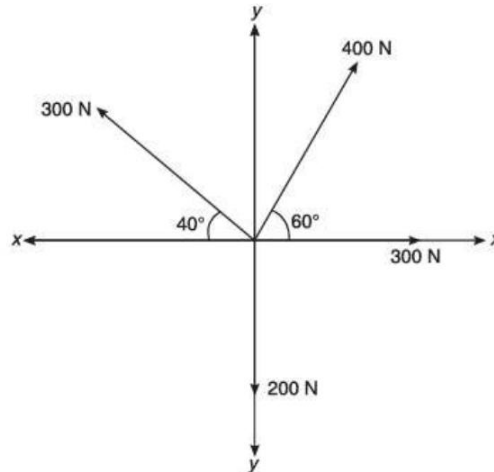


Figure 3.22: Problem 3.

Solution Here:

$$\Sigma F_x = 300 \cos 0^\circ + 400 \cos 60^\circ - 300 \cos 40^\circ$$

$$= 300 + 200 - 229.8133 = 270.187 \text{ N}$$

$$\Sigma F_y = 400 \sin 60^\circ + 300 \sin 40^\circ - 200$$

$$= 346.4102 + 192.8363 - 200 = 339.246 \text{ N}$$

$$\therefore R = \sqrt{(270.187)^2 + (339.246)^2} = 433.692 \text{ N}$$

Ans.

$$\text{Also, } \theta = \tan^{-1} (339.246 / 270.187)$$

$$\therefore \theta = \tan^{-1}(1.256) = 51.47^\circ$$

Ans.

Problem 4: Find the magnitude and direction of the resultant of the coplanar force system shown in Figure 3.23.

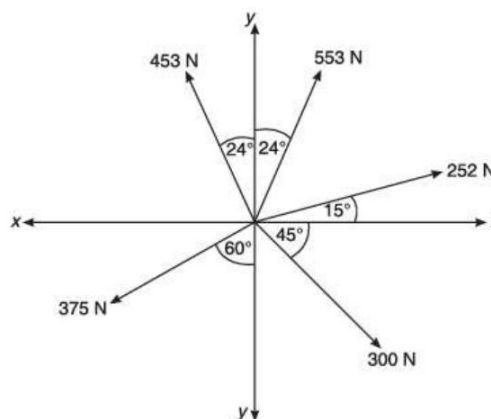


Figure 3.23: Problem 4.

Solution Here:

$$\Sigma F_x = 300 \cos 45^\circ - 453 \sin 24^\circ + 252 \cos 15^\circ + 553 \sin 24^\circ - 375 \sin 60^\circ = 171.459 \text{ N}$$

$$\Sigma F_y = -300 \sin 45^\circ + 453 \cos 24^\circ + 252 \sin 15^\circ + 553 \cos 24^\circ - 375 \cos 60^\circ = 584.617 \text{ N}$$

$$\therefore R = \sqrt{((171.459)^2 + (584.617)^2)} = 609.241 \text{ N} \quad \text{Ans.}$$

$$\text{Also, } \theta = \tan^{-1} (584.617 / 171.459)$$

$$\therefore \theta = 73.65^\circ \quad \text{Ans.}$$

Problem 5: Find the resultant of the force system shown in Figure 3.24.

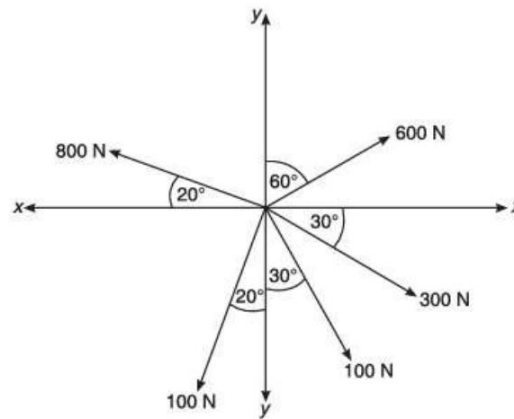


Figure 3.24: Problem 5.

Solution Here:

$$\Sigma F_x = 600 \sin 60^\circ + 300 \cos 30^\circ + 100 \sin 30^\circ - 100 \sin 20^\circ - 800 \cos 20^\circ = 43.467 \text{ N}$$

$$\Sigma F_y = 600 \cos 60^\circ - 300 \sin 30^\circ - 100 \cos 30^\circ + 100 \cos 20^\circ + 800 \sin 20^\circ = 243.044 \text{ N}$$

$$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(43.467)^2 + (243.044)^2}$$

$$\text{or } R = 246.900 \text{ N} \quad \text{Ans.}$$

$$\text{Also, } \theta = \tan^{-1} (\Sigma F_y / \Sigma F_x) = \tan^{-1} (243.044 / 43.467)$$

$$\therefore \theta = 79.86^\circ \quad \text{Ans.}$$

Problem 6: Four coplanar forces acting at a point are shown in Figure 3.25. One of the forces is unknown and its magnitude is shown by P . The resultant has a magnitude of 500 N and is acting along the x -axis. Determine the unknown force P and its inclination with the x -axis.

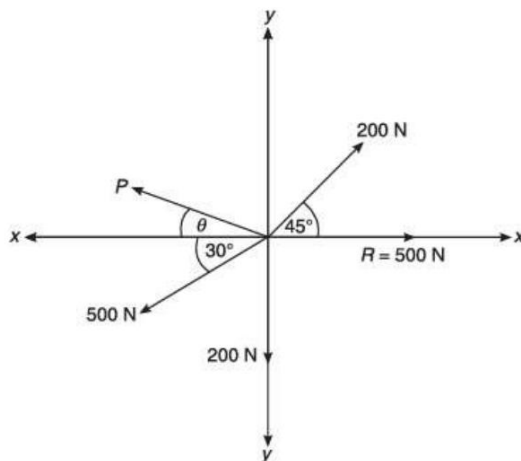


Figure 3.25: Problem 6.

Solution We know that

$$\Sigma F_x = R_x ; \Sigma F_y = R_y$$

Resolving forces along the x -direction,

$$\Sigma F_x = R_x = R \cos \theta = R$$

$$\text{or } \Sigma F_x = 500 \text{ N}$$

$$\text{or } -P \cos \theta + 200 \cos 45^\circ - 500 \cos 30^\circ = 500$$

$$\text{or } -P \cos \theta + 291.591 = 500$$

$$\text{or } P \cos \theta = -791.59 \text{ N} \quad \dots (i)$$

$$\text{Also, } \Sigma F_y = R_y = 0$$

$$\text{or } P \sin \theta + 200 \sin 45^\circ - 500 \sin 30^\circ - 200 = 0$$

$$\text{or } P \sin \theta - 308.579 = 0$$

$$\text{or } P \sin \theta = 308.579 \quad \dots (ii)$$

Squaring both equations (i) and (ii) and then adding,

$$P^2 \cos^2 \theta + P^2 \sin^2 \theta = (-791.591)^2 + (308.579)^2$$

$$\text{or } P^2 = 721837.31$$

$$\text{or } P = 849.61 \text{ N}$$

Ans.

Dividing Eq. (ii) by Eq. (i) gives

$$\sin \theta / \cos \theta = \tan \theta = 308.579 / -791.591$$

$$\text{or } \theta = \tan^{-1} (308.579 / -791.591) = 21.297^\circ$$

Ans.

Problem 7: 26 kN force is the resultant of the two forces, one of which is as shown in Figure 3.26. Determine the other force.

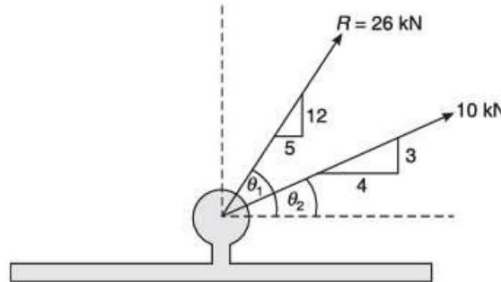


Figure 3.26: Problem 7.

Solution Let P be the unknown force, which makes an angle θ with the horizontal.

Here: $R = 26 \text{ kN}$

$$\theta_1 = \tan^{-1} (12/5) = 67.38^\circ$$

$$\theta_2 = \tan^{-1} (3/4) = 36.87^\circ$$

$$\Sigma F_x = R_x$$

$$\text{or } P \cos \theta + 10 \cos 36.87^\circ = 26 \cos 67.38^\circ$$

$$\text{or } P \cos \theta = 26 \cos 67.38^\circ - 10 \cos 36.87^\circ = 2 \quad \dots (i)$$

$$\text{Also, } \Sigma F_y = R_y$$

$$\text{or } P \sin \theta + 10 \sin 36.86^\circ = 26 \sin 67.38^\circ$$

$$\text{or } P \sin \theta = 26 \sin 67.38^\circ - 10 \sin 36.86^\circ = 18 \quad \dots (ii)$$

Dividing Eq. (ii) by Eq. (i) gives

$$P \sin \theta / P \cos \theta = 18 / 2$$

$$\text{or } \tan \theta = 9$$

$$\therefore \theta = \tan^{-1} (9) = 83.66^\circ$$

Ans.

Squaring (i) and (ii) and then adding

$$P^2 \sin^2 \theta + P^2 \cos^2 \theta = 4 + 324 = 328$$

$$\text{or } P^2 = 328$$

$$\therefore P = 18.11 \text{ kN}$$

Ans.

Problem 8: The resultant of a force system on a bracket as shown in Figure 3.27 is acting vertically upwards. If the angle 30° between the two 4 kN forces is fixed; find the angle θ . Also determine the magnitude of the resultant.

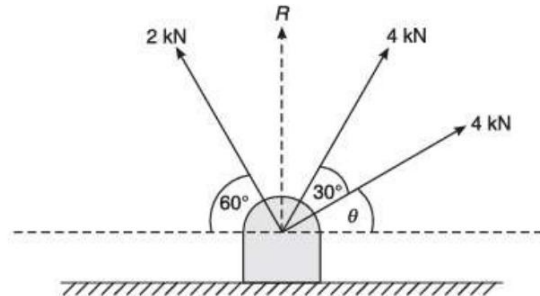


Figure 3.27: Problem 8.

Solution

$$\Sigma F_x = R_x = 0 ; \quad \Sigma F_y = R_y = R$$

$$\Sigma F_x = 0$$

$$\text{or } 4 \cos \theta + 4 \cos(30 + \theta) - 2 \cos 60^\circ = 0$$

$$\text{or } 4 \cos \theta + 4 \cos(30 + \theta) = 2 \times 1/2$$

$$\text{or } \cos \theta + \cos(30 + \theta) = 1/4$$

$$\text{or } \cos \theta + \cos 30^\circ \cdot \cos \theta - \sin 30^\circ \cdot \sin \theta = 1/4$$

$$2 \cdot \cos(15^\circ + \theta) \cdot \cos 15^\circ = 1/4$$

$$2 \times \cos(15^\circ + \theta) \times 0.966 = 1/4$$

$$\cos(15 + \theta) = 1 / (2 \times 0.966)$$

$$15 + \theta = \cos^{-1} (1 / (2 \times 0.966)) = \cos^{-1} (0.517) = 67.57^\circ$$

$$\theta = 67.57^\circ$$

$$\text{Also, } \Sigma F_y = R$$

$$\therefore R = 4 \sin 67.57^\circ + 4 \sin 97.57^\circ + 2 \sin 60^\circ = 9.394 \text{ kN}$$

Ans.

Problem 9: Four forces acting on a hook are shown in Figure 3.28. Determine the direction of the force 150 N such that the hook is pulled in the x -direction. Determine the resultant force in the x -direction.

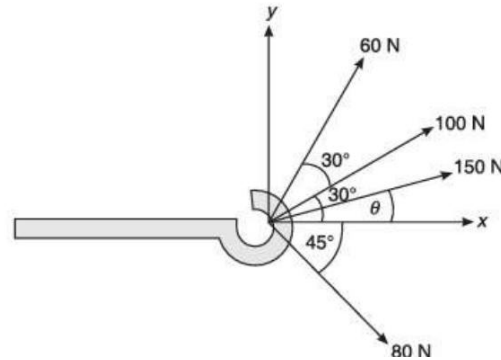


Figure 3.28: Problem 9.

Solution

$$\Sigma F_x = R ; \quad \Sigma F_y = 0$$

For $\Sigma F_y = 0$, we have

$$-80 \sin 45^\circ + 60 \sin 60^\circ + 100 \sin 30^\circ + 150 \sin \theta = 0$$

$$\text{or } 150 \sin \theta = 45.39$$

$$\text{or } \theta = \sin^{-1} (45.39 / 150) = 17.61^\circ$$

For $\Sigma F_x = R$, we have

$$80 \cos 45^\circ + 60 \cos 60^\circ + 100 \cos 30^\circ + 150 \cos 17.61^\circ = R$$

$$\therefore R = 316.142 \text{ N}$$

Ans.

Problem 10: Four forces are acting on a bolt as shown in Figure 3.29. Determine the magnitude and direction of the resultant force.

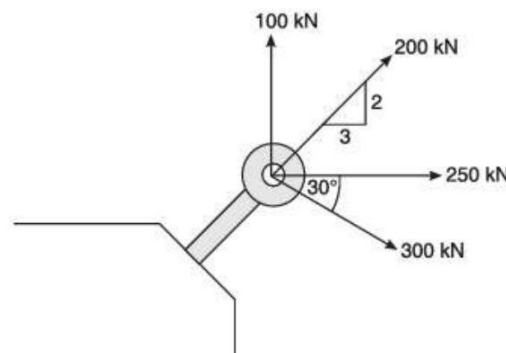


Figure 3.29: Problem 10.

Solution Magnitude of the resultant,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\Sigma F_x = 250 + 300 \cos 30^\circ + 200 \cos 33.69^\circ = 676.218$$

$$\Sigma F_y = 100 - 300 \sin 30^\circ + 200 \sin 33.69^\circ = 60.939$$

$$R = \sqrt{((676.218)^2 + (60.939)^2)} = 678.958 \text{ kN} \quad \text{Ans.}$$

$$\text{Direction of resultant } (\theta), \theta = \tan^{-1} (\Sigma F_y / \Sigma F_x)$$

$$\theta = \tan^{-1} (60.939 / 676.218) = 5.149^\circ \quad \text{Ans.}$$

Problem 11: Two forces of 800 N and 600 N act at a point as shown in Figure 3.30. The resultant of the two forces is 1200 N. Determine θ between the forces and the direction of the resultant.

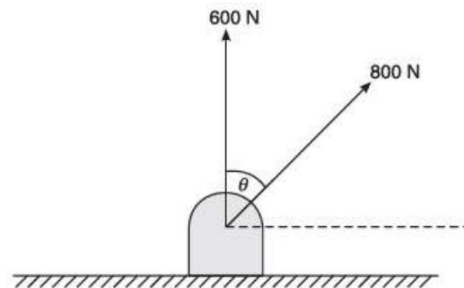


Figure 3.30: Problem 11.

Solution

$$\Sigma F_x = 800 \sin \theta$$

$$\Sigma F_y = 600 + 800 \cos \theta$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\text{or } (1200)^2 = (800)^2 \sin^2 \theta + (600)^2 + (800)^2 \cos^2 \theta + 2 \times 800 \times 600 \times \cos \theta$$

$$\text{or } (12)^2 = 64 + 36 + 2 \times 8 \times 6 \cos \theta$$

$$\text{or } 144 = 100 + 96 \cos \theta$$

$$\text{or } 44 = 96 \cos \theta$$

$$\text{or } \cos \theta = 44/96$$

$$\therefore \theta = \cos^{-1} (44/96) = 62.72^\circ \quad \text{Ans.}$$

From the relation, $\tan \phi = Q \sin \theta / (P + Q \cos \theta)$, we have

$$\tan \phi = 800 \sin 62.72^\circ / (600 + 800 \cos 62.72^\circ) = 0.736$$

$$\text{or } \phi = 36.34^\circ \text{ with respect to the vertical}$$

$$\therefore \text{The direction of the resultant with respect to the horizontal} = 90^\circ - 36.34^\circ = 53.66^\circ \quad \text{Ans.}$$

Problem 12: Determine the resultant of the system of forces acting on a body as shown in Figure 3.31. Take the co-ordinate directions as shown in the figure.

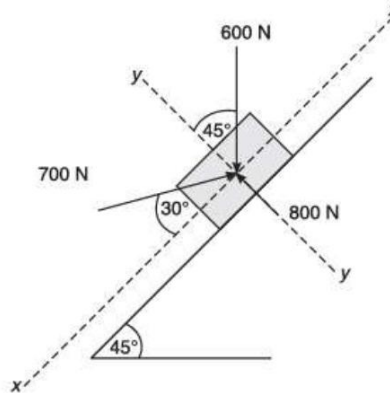


Figure 3.31: Problem 12.

Solution

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\Sigma F_x = 700 \cos 30^\circ - 600 \sin 45^\circ = 181.953 \text{ N}$$

$$\Sigma F_y = 800 - 600 \cos 45^\circ - 700 \sin 30^\circ = 25.736 \text{ N}$$

$$\therefore R = 183.764 \text{ N}$$

$$\text{and } \theta = 8.05^\circ$$

Ans.

Ans.

Problem 13: A force of 200 N is acting on a block as shown in Figure 3.32. Find the components of the force along the horizontal and vertical axes.

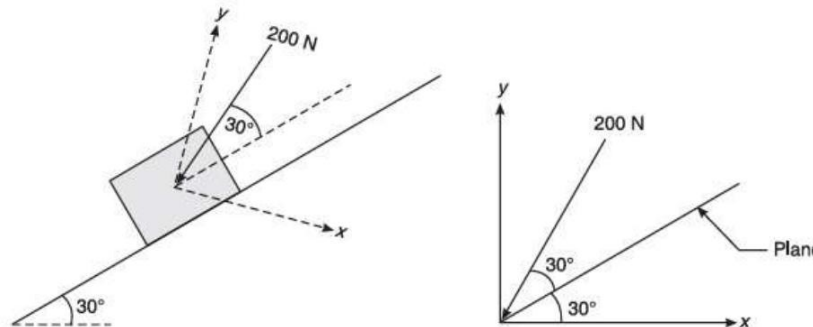


Figure 3.32: Problem 13.

Solution

$$F_x = -200 \cos 60^\circ = -100 \text{ N}$$

$$F_y = -200 \sin 60^\circ = -173.2 \text{ N}$$

Ans.

Ans.

Exercise Problems

1. Determine the resultant of the four forces acting on a particle as shown in Figure 3.33.

[Ans.: $R = 303.4 \text{ N}$, $\theta = 68.85^\circ$]

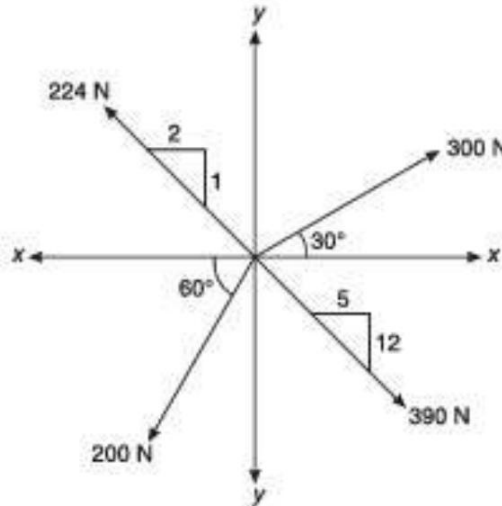


Figure 3.33: Exercise 1.

2. Determine the resultant of the coplanar concurrent force system shown in Figure 3.34.

[Ans.: $R = 155.81 \text{ N}$, $\theta = -76.64^\circ$]

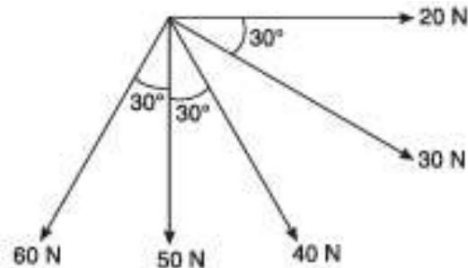


Figure 3.34: Exercise 2.

3. Determine the resultant of the force system shown in Figure 3.35.

[Ans.: $R = 104.84 \text{ kN}$, $\theta = -11.52^\circ$]

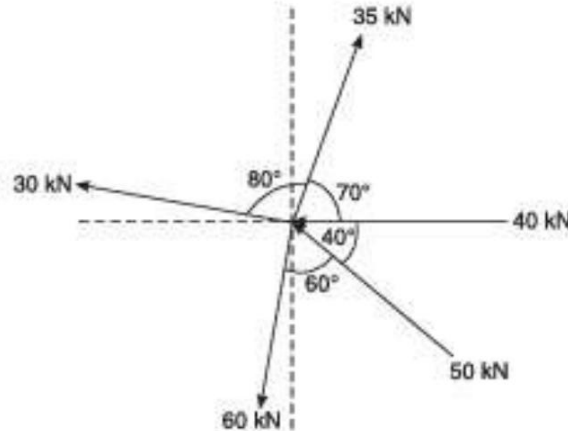


Figure 3.35: Exercise 3.

4. The force system as shown in Figure 3.36 has a resultant of 240 N acting up along the y -axis. Compute the value of force P and its inclination with x -axis.

[Ans.: $P = 151.16 \text{ N}$, $\alpha = 74.51^\circ$]

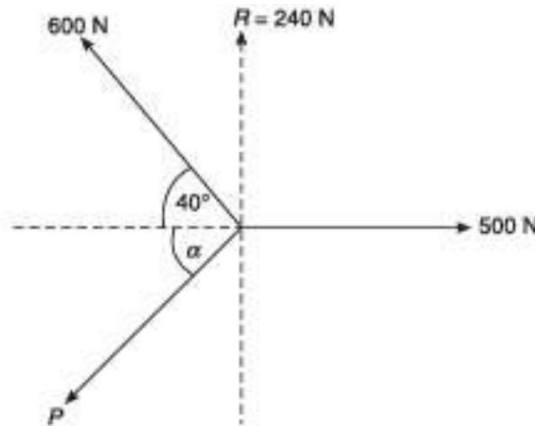


Figure 3.36: Exercise 4.

5. The following forces, as shown in Figure 3.37, are acting at point A. Determine the resultant.

- A force of 200 kN directed towards 30° of east of south.
- A force of 100 kN directed towards 60° of west of north.
- A force of 300 kN directed towards 40° of west of south.
- A force of 50 kN directed towards north.
- A force of 150 kN directed towards west.

[Ans.: $R = 447.604 \text{ kN}$, $\theta = 42.61^\circ$]

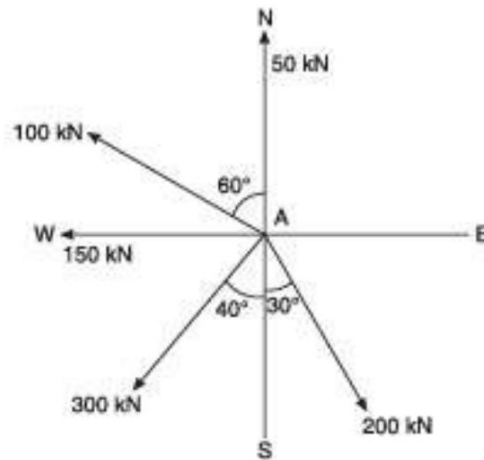


Figure 3.37: Exercise 5.

6. The two forces P and Q are acting on a bolt at A, as shown in Figure 3.38. Determine the magnitude and direction of the resultant.

[Ans.: $R = 91.23 \text{ N}$, $\theta = 67.97^\circ$]

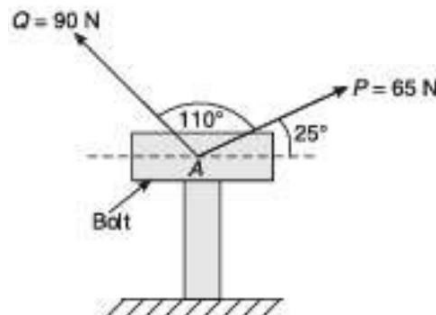


Figure 3.38: Exercise 6.

7. Figure 3.39 shows the top view of a car, pulled by two cables AB and AD. The car is moving along AC. If the force in cable AB is 100 N, calculate the force in AD and the resultant.

[Ans.: $F_{AD} = 70.711 \text{ N}$, $R = 136.603 \text{ N}$]

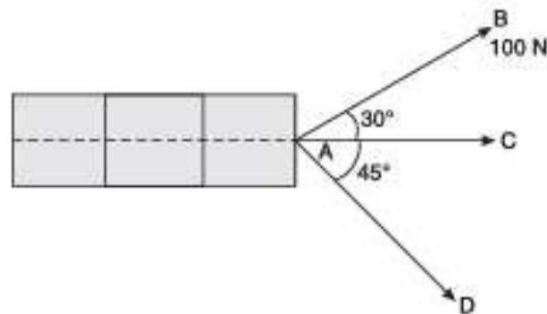


Figure 3.39: Exercise 7.

8. The 26 kN force is the resultant of two forces, one of the forces 10 kN is shown in Figure 3.40. Solve for the other force, F .

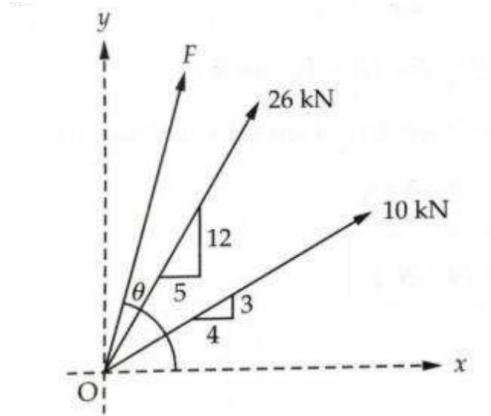


Figure 3.40: Exercise 8.

9. Solve for the magnitude and direction of the resultant of the four forces acting on a particle as shown in Figure 3.41.

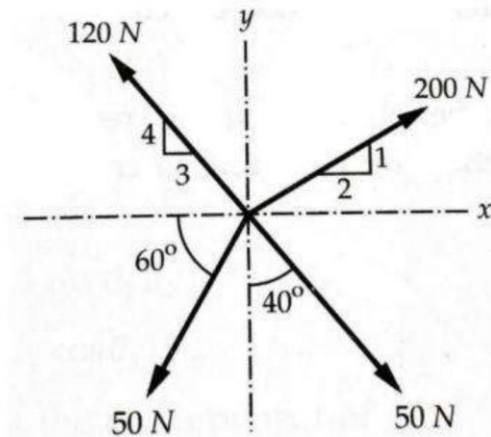


Figure 3.41: Exercise 9.